Making Math

Developing Student Mathematicians

When I was a calculus student in high school, I had fast and accurate technical skills. But if you asked me what 7 x 8 was, I would answer 56 readily, but I was also checking it in my head: *4 times 7 is 28, 28 doubled is 56, good, yes, 56*. I still, at 18 years old, did not trust my answers to 7 x 8, 6 x 8, or 6 x 7. But it was my secret, because I knew how to rederive those number facts quickly from smaller ones about which I was more confident. Thoughtful contemporary curricula help students build up a number of understandings and strategies that support mastering arithmetic facts (e.g., learning number pairs that sum to 10 and then how to use them for regrouping and finding other sums). However, too many parents, students, and traditional teachers see number facts as something to memorize and repeat quickly rather than pieces of information that have a relationship to each other, that they are interconnected facts.

Unfortunately, after teaching content through understanding, that understanding is too frequently abandoned in favor of memorization. When students are taught that it is time to memorize, they often move on from the understanding behind those facts and that leaves a rather shaky foundation (If a fact is only memorized and not connected to other facts, students have fewer ways to check or reconstruct it when memory fails). For example, The Common Core Standards present different approaches to understanding whole number multiplication, but then call for students to, “by the end of grade 3, know from memory all products of two one-digit numbers.” The danger is that “know from memory” is too often interpreted as “stop reasoning and start parroting.” But memory built through relationships is more durable than memory treated as distinct tasks. If we avoid framing math as a speed sport and instead give students time to practice rederiving knowledge, then memorization can emerge from those processes repeated in the service of interesting tasks.

The goal is not to prevent students from memorizing facts. The goal is to make sure that memorization is the result of repeated reasoning, pattern recognition, and use—not a substitute for them.

Student Activities

The two activities shared here are modest in format, but I have used them effectively with late elementary and middle school students who need more practice understanding, deriving, and retrieving multiplication facts.

1. The first is a handout with many, but not all, single-digit multiplication problems listed. Students are not asked what the products are; they are asked how the products in different circles relate. The layout could definitely be improved (mostly by having more free space for connecting arrows to be drawn perhaps by using legal or larger paper). Students are asked to find products that relate through multiples: "can you draw a connecting arrow between two circles where the one being pointed to is <u>twice as large as </u>the one where the arrow started?" The underlined part can be replaced with half as much, three times as much, 7 more than, etc. Students should draw an arrow and label it with the operation that converts one product into the other. For example, 9 x 6 can be found by starting with an easier product, 10 x 6, and subtracting 6. After a few examples are worked on and discussed, students can be set loose finding as many different kinds of relationships and adding them to their diagram (extra copies are helpful once a diagram starts to get cluttered). They can even try to randomly point to pairs of circles and see if they can come up with a meaningful relationship (aside from just calculating the two products and saying something like "add 34"). Useful prompts include:
    - Can you find another pair with the same relationship?
    - Can two products be related in more than one way?
    - Can you make a chain of three related products?
2. The second, Factor Finder, is an activity where students see a blank multiplication table with only the axes labeled. They are given a product and asked to select the square in the table that produces it. Because of the reverse nature of this process, this game links division and multiplication.
   When a student completes the final square for a given product (e.g., selects 8x3, when 3x8,4x6, and 6x4 have already been chosen), all squares that produce the same result our outlined and pulse. This illustrates that equal products are arranged in a structured way along an arc. This is an opportunity to imprint a bit the shape of the hyperbolas that will arise when they study inverse variation, $y = {}^{k}/_{x}$. We get the same graph when studying $xy = 24$ as we do with $y = {}^{24}/_{x}$.
   Students can build up from smaller multiplication tables to bigger ones as their mastery evolves. They can also start with the untimed version and then, when they feel ready and are making few, if any, mistakes, use the timed mode. They still have as much time as they need, but they can then start trying to beat their own best prior effort (not

anyone else's time, which is not relevant to where they are in their learning). Students should not be pushed to treat speed as the main goal (that suggests that time for thought should be avoided and we want them to be in the habit of sitting to a task and thinking patiently), but some students will definitely be motivated to improve their performance in ways that will lead to greater fluency.

These activities are meant to keep facts connected. A student who knows that 7×8 is 56 is in good shape; a student who can also recover that fact from 7×4, $5 \times 8 + 2 \times 8$, $8 \times 8 - 8$, or $10 \times 7 - 2 \times 7$ has something even stronger than recall. They have a network of information based on understanding.