Making Math

Developing Student Mathematicians

Introduce algebra with expressions!

Many students' first experience with algebra comes in the guise of arithmetic practice. For example, they are given problems such as 7 + $\square$ = 22 and asked to fill in the boxes. At some point, a question like this morphs to 7 + *x* = 22. This switch in notation raises a number of issues. One, the unknown, *x*, no longer appears to be a container and students typically call it a "letter" rather than something they imagine is a placeholder for a value. The next issue is that this new way of representing arithmetic does not typically connect to any new interesting use. Solving equations as a first algebraic experience teaches that algebra is about "finding the one right answer" – an answer the teacher and textbook already know. These one-variable, linear equations involve an *unknown* with a single solution. The unknown is not a variable that can *usefully* take on many values. Yes, we can plug in whatever we want, but only one value solves the equation.

Starting early algebra exposure with expressions makes it possible to introduce applications and engaging problems that highlight algebra's power. Expressions represent an infinite number of possibilities all of which have meaning. They can represent and explain a whole set of problems all at once. That is the power of algebra we want to wow students with at the start!

First class activity: **Mind Reading**. This teacher-led class "magic trick" is explained through the use of expressions.

Fingers – This follow-up class discussion reinforces that a variable is a place holder for all possible values at once. Ask everyone in a class to write down how many fingers are in the room (strangely, students often count how many kids are in the classroom and ignore the teacher's fingers). Get agreement on the correct response. Now ask how many there would be if 6 more people entered the room. Then ask how many there would be if Baseball Hall of Fame pitcher Three-finger Mordecai Brown (from the early 20th century) entered. This is a slightly trick question

© 2025 by Joshua Abrams/Making Math and licensed under CC BY-NC-SA 4.0

since Brown had three fingers on his pitching hand due to a childhood farming accident, but had 8 fingers overall. Now ask the same questions, but with variables, reminding the students to do the exact same operations as they did before. How many fingers are there if there are *S* students in the room (will they remember your fingers this time?). If *N* additional students enter? If 2 students leave? If *L* students leave? If Mordecai Brown enters? What if a three-fingered sloth enters (note, they have four limbs with three fingers, or toes?, each). What if *T* of them enter?

Expression Practice 1 – This in-class or homework set of problems, inspired by Harold Jacob's *Elementary Algebra* text, moves students from concrete examples to abstract ones. The key message is that whatever operations they do for the first examples in each cluster of problems are the same ones they should do for the general cases.

Expression Practice 2 – These problems focus on substituting in values to explore specific cases for the sets of expressions and also having students look at the symbols to consider what they have to do with each other. On the second page is a thought bubble saying "Shoes and socks". In the toothpick problem set (read below), there is an odd doodle. For those of you familiar with Ben Orlin's *Math with Bad Drawings*, I think I set the standard for truly bad drawings long before. That doodle is a picture of a foot in a sock (the blue squiggle) inside of a shoe (the boxy olive green shape). Shoes and socks are a reminder of how one solves equations by undoing operations in the reverse order that they act on a variable. I ask students what they do with regard to their feet when getting dressed in the morning. They say, of course, that they put their socks on first. And then I ask what they do when they come home. Do they take their socks off first? Why, no. That would be problematic. They have to undo the layers in the reverse order that they put them on (if you need a third layer for an example with three operations, you can always include rubber rain overshoes, known by the funny old-fashioned word "galoshes"). So shoes and socks are a visual model for the order of operations. For great text materials on solving equations with this idea in mind, consult *Impact Mathematics, Course 2* (used copies are available very affordably online). Chapter One introduces expressions and how to represent them with flow charts and Chapter Six teaches how to solve equations by working backwards through the flow charts.

© 2025 by Joshua Abrams/Making Math and licensed under CC BY-NC-SA 4.0

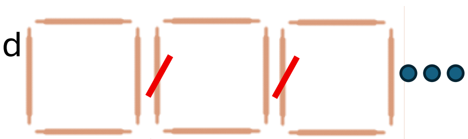
Toothpick Problems – This geometric setting produces a nice variety of approaches. Student expressions for the number of toothpicks needed for *n* squares may suggest some of these:

$3n + 1$ looking at the squares as a sequence of nested C's with a final closing toothpick.

$2n + n+1$ seeing *n* toothpicks above and *n* below and *n*+1 vertical toothpicks.

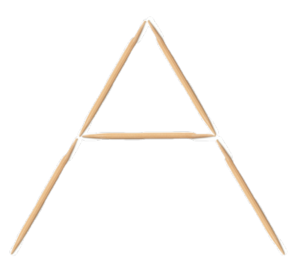
$4 + 3(n – 1)$ representing an initial full square and additional backward C's completing each subsequent square.

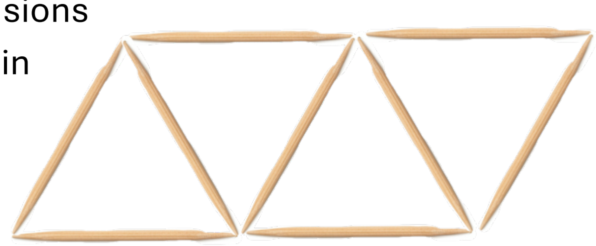
$4n – (n – 1)$ counting 4 toothpicks for each square and removing the duplicates for shared edges.

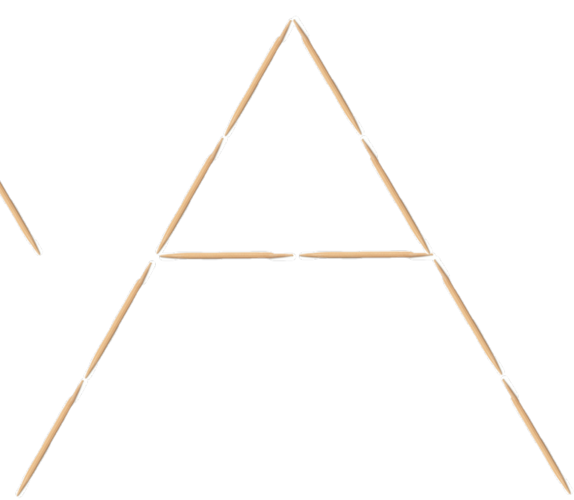


Note: students are likely to have explicit multiplication signs and those are fine. Implied multiplication can wait if it does not come up. These different representations are a great opportunity to talk about equivalent expressions. First, have students plug in different values for *n* and see that they all work. Then, the class can discuss distribution for the third model and like terms for the final two. The idea that $2n + n$ equals $3n$ can be based on students' notions that we are simply counting, and that is what the idea of like terms entails: we are seeing how many times we have a particular value (not a particular symbol).

The final pages on grids of squares and grids of toothpick cubes are extensions. The cubes are probably best for students who want extra challenging problems, but most students who come up with a representation for the line of squares will be able to come up with a reasonable approach for the grid. It is a nice example of how we can produce new problems by simply adding dimensions to familiar problems in one or two dimensions.

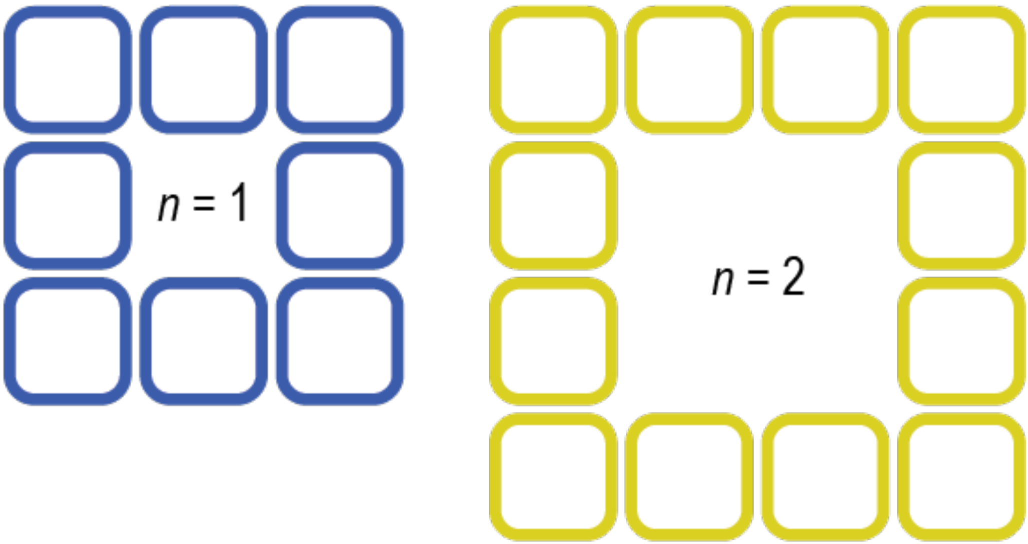


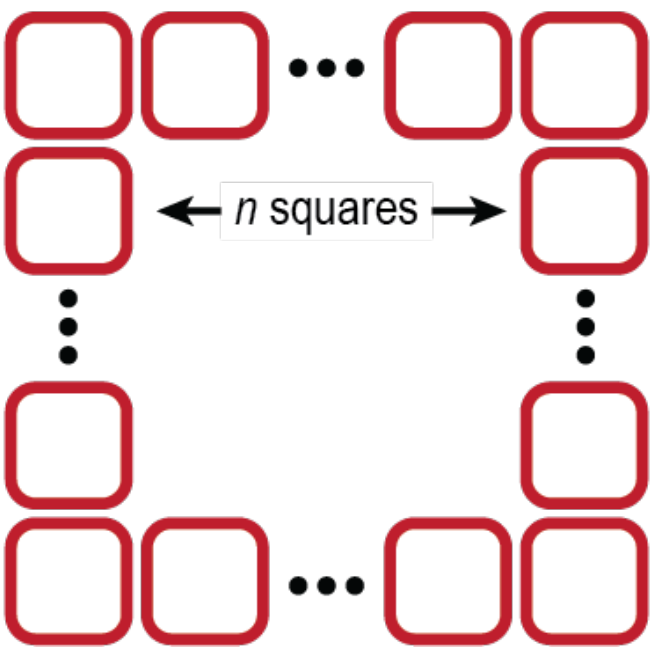




© 2025 by Joshua Abrams/Making Math and licensed under CC BY-NC-SA 4.0

Having a box of actual toothpicks (flat, not round, so they don't roll) makes it easier for students to come up with their own problems. Possibilities include triangles or a scaling series of one of their initials (see prior page). Other manipulatives, such as unit square tiles, or other shapes, can produce similarly interesting alternative solutions (and it is always great when kids see numerous ways to solve the same problem). For example, the number of tiles in a square frame for an opening of *n* by *n* tiles can be counted in many ways:





© 2025 by Joshua Abrams/Making Math and licensed under [CC BY-NC-SA 4.0]

ADDITIONAL SUGGESTIONS

Programming – There are no activities posted here (for now) about introducing computer programming, but it is worth noting that kids love learning basic coding and seeing how a computer can be controlled. Importantly, any program much beyond the classic first program (print("hello world!") involves the use of variables (as variables). Simple input/output programs that ask the user for some data and then perform calculations on it involve algebraic expressions that solve an infinite set of problems. Some simple examples involve unit conversions (e.g., ask for a length in inches and report how many feet or miles it is; ask someone their age and tell them how many seconds they have been alive). Once students know conditional (if-then) statements, they can do different tasks depending on a variable's value (e.g., making a program that turns words into Pig Latin or that says how much they earn with overtime beyond a certain number of hours worked). Both Scratch and Python are fine tools for these activities and also support great animation projects that build on algebraic tools as well.

Computer-Aided Design (CAD) – Materials for teaching geometry and algebra with CAD are in Making Math's Engineering resources. Many schools, especially in middle school STEM classes, introduce engineering with CAD programs. Often, they pick ones that are closer to 3D drawing programs than true CAD tools inasmuch as they lack some of the power that makes learning CAD a great complement to algebra and geometry studies. OnShape is a powerful professional CAD program that is also accessible to middle schoolers and allows the use of variables and more complicated algebraic expressions to represent lengths within a design. Making shapes that change according to these parameters shows students the power of variables in a new setting. They have not just designed one shape, they have designed a family of related shapes (perhaps all geometrically similar or related in other ways). OnShape is free (until you want to keep all of your designs secret, which is not the goal for a class, so it is a great option).

© 2025 by Joshua Abrams/Making Math and licensed under CC BY-NC-SA 4.0