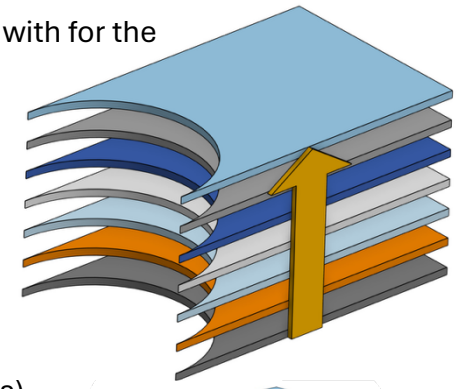
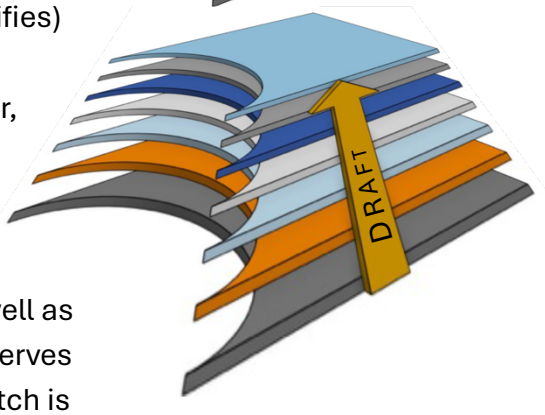
Making Math

Developing Student Mathematicians

Now that students have a range of sketching skills, it is time to combine them with the tools that move sketches through space to make three-dimensional forms. The two most basic 2-D to 3-D tools are **extrude**, which the students have already worked with for the brick and initials cube projects, and **revolve**.

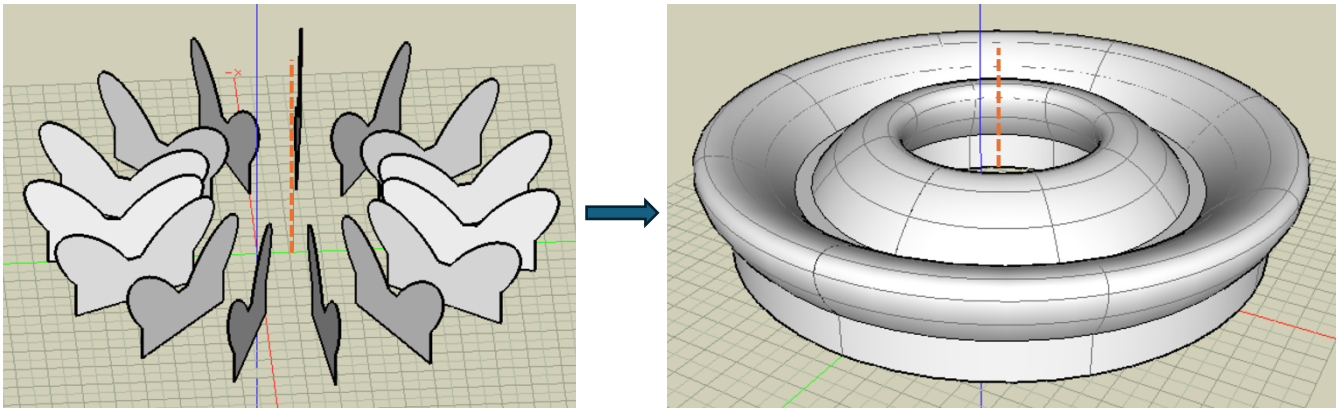


For an extrusion, we start with a closed figure (holes allowed) and move it perpendicularly to the plane leaving copies of the sketch in its wake. Extrusions are like the image at right, but with every gap filled in with an infinite number of *congruent cross-sections parallel to the original base*. Extrusions make prism-like solids.

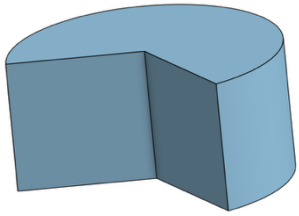


One can also extrude with a **draft**, which scales (shrinks or magnifies) the cross-sections proportionally throughout the distance of the extrusion. In this case, each cross-section is geometrically similar, but not congruent, to the original. The amount of scaling is controlled by the **draft** – the angle at which the surface is bent inward or outward from the base.

A revolution also begins with any type of well-defined sketch as well as an edge of the shape or line segment external to the sketch that serves as the **axis of rotation** (dashed orange segments below). The sketch is then spun around that axis leaving congruent copies as it goes.



If the axis of rotation does not intersect the sketch, as in the example above, the resulting shape will have a hole. The axis cannot be internal to the shape; it can only be an external edge or non-intersecting line. Internal axes lead to self-intersecting shapes and OnShape will not generate a figure. Rotations can be less than 360° like a cake with a missing wedge.

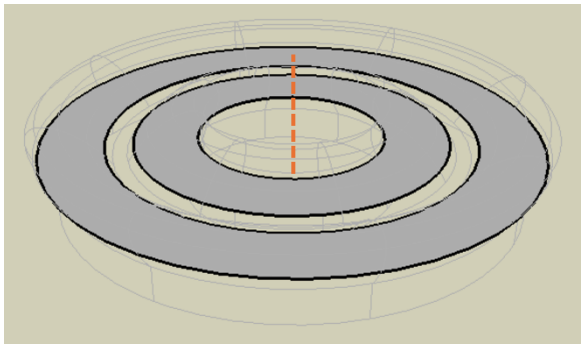


How to tell when an object can be made by extruding or revolving

I like to use the word "diagnosis" with students when it comes to effective problem solving. We don't know what steps to take until we ask relevant diagnostic questions and see what answers we get. To diagnose a case of extrusion (which is not catching), we have to see these features:

- A flat (planar) face of the shape
- A parallel face that is congruent or similar to the first face, or a single point. If a point, we may have a pyramid-like or cone-like shape that can be an extrusion with a draft that comes to a vanishing point.
- All of the cross-sections between the two above faces must be parallel to those faces, congruent to them, or a scaled (similar) version that changes scale linearly from one face to the other. The cross-sections can't bulge in and out for an extrusion (in this case, you would need a loft, which is described below). For example, the cross-sections of the table leg at right are all circles, but that is not an extrudable object.

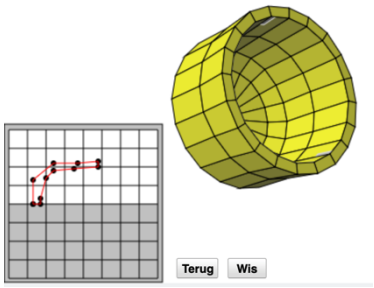
Solids of revolution are a little trickier. It is not too hard to identify them from their roundness, but visualizing the cross-sections takes some practice. Here are our questions for a shape made by a 360° revolution:



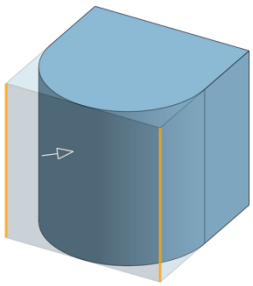
- Is there a direction in which the cross-section is a circle, a ring, or multiple concentric rings? The example at right is for the shape on the prior page.
- Are all cross-sections parallel to the one above also a circle or ring(s)?
- Locate the centers of these circles and draw a line through them. This line is perpendicular to the planes of the cross-sections. If you picture making a "cake cut" from that center line outward in all directions and all of the surfaces that these cuts expose are congruent, then that is the shape being revolved around the center line.

Activity 1: Distribute the Volumes of Revolution handout and demonstrate it for the class. The instructions are in English (if not, back up to the main Applet menu, https://app.dwo.nl/wisweb/?header=less&hash=#s:603082, and click on the little British flag at the top of the window and then return to the Lathe applet), but the buttons are not. "Terug" means back up one step (it undoes the last segment added) and "Wis" means clear and gives a fresh lathe window. To use the applet, students click and draw segments in the white upper region, and those segments are revolved around the line separating the white and black areas of the grid. The resulting three-dimensional shape is shown and can be rotated and examined by clicking and dragging near the shape.

Students should first just play around and get used to the tool and then try to create each of the suggested shapes. At right is a cross section that, when revolved around the axis of rotation makes a cup.



Some shapes can be made in more than one way



Work with CAD designs provides a great opportunity to remind students that there is not just "one right answer." There are often multiple paths to the same final form. Even something as simple as a cylinder can be made in at least three ways: extruding a circle, revolving a rectangle about one of its edges, or making a box and then filleting four edges with a radius half as wide as the box (as shown in process at right).

When more complex forms are tackled, there will be numerous sequences that get the job done. Sometimes, a student may build up a shape in steps with the extrude "Add" option or make a shape and then "Remove" unwanted regions. For example, for the Lego-style brick activity, we removed the inner region, but could have instead started with a relatively flat top surface and then added the four walls.

The message about multiple good solutions extends when students are designing their own engineering solutions to problems – there are always many possibilities with different trade-offs. Math, science, and the world are nuanced!

Here are several instructional videos that may be helpful for you and/or your students.

Extrusion (with draft)

- The Teach Product Design (TPD) video series has many great OnShape videos: https://www.youtube.com/playlist?list=PLXujnhvVBY9i7odsdapiqAu_LZwCVUy7v. Scroll all the way down the list to four beginning extrude videos. At the top of the list are two more useful extrude videos (first here and second here) that will broaden your toolkit for ways to use extrude.
- Here is a TPD video on using drafts with extrudes.
- Onshape's extrude video is here.

Revolution

- Here is TPD's introduction to revolutions lesson and a more advanced video here.
- Here is OnShape's introduction to revolutions.

Activity 2: OnShape's Section View Tool. OnShape has a great tool that helps students further explore cross-sections of shapes. Show the Onshape's Sections View Demo video or do a demo yourself and encourage students to make some 3-D shapes or use previous ones that they have created and practice with the tool. Have students demonstrate in OnShape examples that fit the diagnosis questions above for an extrusion and revolution to a partner.

Activity 3: Revolve and Extrude Lab.

- Have an objects table set up in advance with a mix of objects that can be made through extrusion, revolution, both, and neither. A box, ice cream cone or cone-shaped paper cup, hemisphere, pentagonal or triangular pyramid, a can or dowel, paper towel tube (with or without the paper), a fairly symmetric carrot, piece of celery, foam packaging, and other objects are all good choices.
- Distribute the Practice Revolutions and Extrusions handout.
- For both the objects pictured in the handouts and real objects, we want to encourage recognizing basic forms even if the match is not perfect. For example, the picture of the cheesecake on the fifth page of the handout can be made with a revolution, but it can also be an extruded shape if we ignore the rounded lip at the top (which could be added afterwards). Some of the balusters have cut-out designs that make them not a perfect volume of revolution, but, again, those can be (and no doubt were) added after they were produced on a lathe. Approximately right is a good start and all solutions that work pretty well should be embraced and compared.

Vocabulary

Activity 4: Geometry studies are typically the most vocabulary rich time in a student's secondary mathematics education. It is helpful to periodically spend some time reviewing the many terms that students are encountering as they learn 2-D and 3-D drawing on paper and in a CAD program. See the Geometry Vocabulary handout and give students time to work in pairs to fill out definitions for each term.