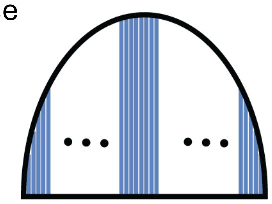
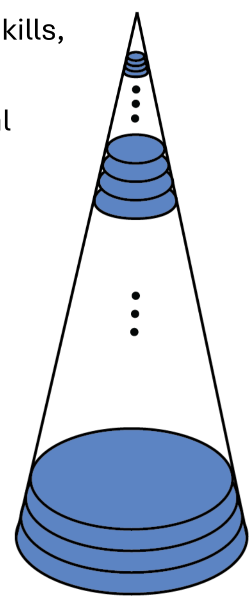
Making Math

Developing Student Mathematicians

Understanding and Creating In Two and Three Dimensions – Geometry and Computer-Aided Design (CAD)

This unit develops students' understanding of two- and three-dimensional shapes; measurement in 1-D, 2-D, and 3-D; and how shapes in a given dimension can be built up from those of the dimension before it. Working with OnShape, a powerful, yet accessible, computer-aided design (CAD) program (that is free for the version we need), gives students the opportunity to practice the above understandings while working creatively with a tool that also encourages precision in thought and measurement. Working with CAD software makes it possible for students to produce tools, works of art, and other objects using computer-aided machines (CAM) such as 3-D printers and laser cutters.





These studies also enhance students' geometry vocabulary and mental imaging skills, such as the ability to rotate, reflect, or scale an object in their mind and to see imbedded figures (shapes that are part of more complex shapes). Stronger mental imagery skills lead to stronger geometry abilities (e.g., seeing that two differently oriented and scaled figures are similar) and nurture intuition about how an infinite number of segments can comprise an area or an infinite number of areas can constitute a volume (see the figure at right). These skills and understandings are all essential to strong calculus mastery but are not developed in typical math classes and are especially absent from the misnamed hodge podge of topics called *Pre*-calculus. This unit is true preparation for calculus thinking! The best time to do these units in middle school or early on in a high school geometry class (or, I suppose, in Precalculus – better late than never).

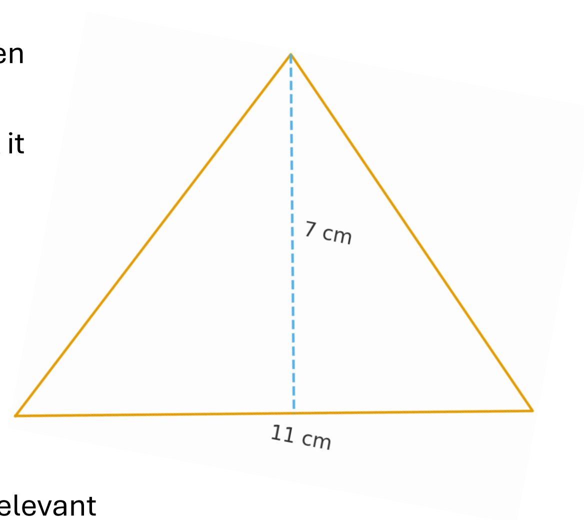
Materials

Throughout this unit, there is a focus on hands-on measurement. Toward that end, a classroom should have enough of the following items for all students.

- Centimeter cubes. The plastic ones are better than wood, because they to have the same density as water 1 gm/cm^3, which makes for interesting science and measurement comparisons. The colors also make distinguishing the unit cubes in larger groupings easier and makes it possible to look at orthographic projects with the extra positional hint of color.

- Metric rulers (more manageable than meter sticks for measuring small objects and on handouts) with both metric and customary (which is a custom we should abandon!) units.
- Meter sticks (not 36" yard sticks that have metric measure). Having many means kids can measure longer distances (e.g., size of the room) readily by laying two sticks end to end and alternating them (or laying lots of sticks end to end and counting them all).
- Graph paper. Especially centimeter paper.
- Calipers. These are optional, but lots of fun and emphasize that precision matters in engineering design. They are particularly helpful for measuring small distances and hard to reach internal distances (they have a depth measuring component). Because of their shape (the pointy face, ears, and long neck, we often call them "giraffes" in class).
- Anglegs. These are great for demonstrating the rigidity of triangles and lack of rigidity of polygons with more than one side. They can be used to understand and practice angle measurement and to understand triangle and other congruence theorems.
- Have a bag of various regular and irregular shapes: boxes, cones (paper cups, cardboard yarn cones), pyramids, shapes made from Polydron, spheres (balls of different sizes, such as marbles, ping pong balls, basketballs, or inflatable beach balls), hemispheres, platonic and other solids (here or here), strange foam packing shapes (this is a great opportunity to look at your garbage with a new opportunistic eye), cylinders (cans, dowels, etc.) of varied scale from pretty small items (such as thumbtack, which is basically two cylinders) to large objects. Keep an eye out in your life for fun objects to add. The point of this variety is to practice recognizing basic forms and then to see them and parts that are approximately like them as part of complex forms. Using real objects means students get to figure out which lengths they need (unlike textbook problems which give them just the lengths they need and no more as in the figure at right).

While working with actual objects is best, note that even when you do give students handouts with images of shapes to find the area and volume for, you should mix it up: give them too many measurements or none at all (they should use a ruler to find the needed lengths). When giving diagrams, don't always orient shapes in the stereotypical "base" down fashion. Rotate triangles, etc., which makes identifying altitudes more revealing in terms of how well students can see and draw perpendicular lines. Diagrams like the one at right don't engage students in actually identifying the relevant measurements themselves.



The core coherent sequence for the measurement and CAD unit is:

- Visualizing 3-D shapes with 2-D drawings: isometric and orthographic diagrams.
- Understanding the different 2-D and 3-D shapes that many objects are made up of. This work includes the definitions of types of triangles, quadrilaterals, and solids.
- The meaning of units: Metric lengths. Segment lengths named after students and made up power of 3 prefixes. Metric prefixes. How units work across the dimensions.
- Basic CAD sketching tools and associated vocabulary (congruent, parallel, perpendicular, concentric, etc.). Measurement and dimensioning.
- Transformations: Translation, rotation, reflection, and scaling. Symmetry. Linear and circular repetitive patterns.
- 3-D tools that turn 2-D cross sections into 3-D solids: Extrusion (with and without draft), Revolution, Lofts, and Sweeps. Note, CAD programs can also generate complex surfaces like the everchanging curves of car bodies, but the basic tools, especially loft, used cleverly, give students plenty of powerful options for a first experience.
- Set operations: intersection, union, subtraction