



Making Math

Developing Student
Mathematicians

Before the project described below, it is good to share with students more about computer-aided manufacturing (CAM) as the next step after CAD design projects. Some classes may have access to a 3-D printer or laser cutter. If so, you can follow this project with CAM projects. Whether or not a class has those tools, it is great to show them some of the myriad ways 3-D printing is being used in medicine, engineering, art, and other fields. A sample of videos is listed below. You can also ask students to pick a hobby or career they are interested in and search for information (search for “3-D printing and _____”). For example, searching for “3-D printing and sports” reveals new types of equipment, and searches with “dentistry”, “plumbing”, and many other careers reveal the impact that CAD and CAM are having. If students do have gain experience with 3-D printers and/or laser cutters in class and are given access afterwards, they are likely to look for opportunities to use CAD in future courses both within STEM subjects (e.g., to make needed items for original experiments) and as a new medium they can use for art projects.

Videos Demonstrating Different Applications of Computer-Aided Manufacturing

- Intro: (PBS Will 3D Printing Change the World?): <http://youtu.be/X5AZzOw7FwA>
- Prosthetic arm for a kid: <https://www.youtube.com/watch?v=N01QN-3OB3s> and <https://www.youtube.com/watch?v=zTBH8jBwJFM>.
- BioPrinting: <http://www.youtube.com/watch?v=Cw1TCPio6Gk>
- More BioPrinting: <http://www.youtube.com/watch?v=bzyXKL9sW70>, <https://www.youtube.com/watch?v=mMeOjwH4NVU&t=85s>, and <https://www.youtube.com/watch?v=INm3Q0xVWrQ>.
- 3-D printed rockets: <https://www.youtube.com/watch?v=kz165f1g8-E>
- Fashion – 3-D printed dresses with different materials. <https://web.archive.org/web/20221125213559/https://www.youtube.com/watch?v=2uhR1NFbtL0>
- Nervous System <http://n-e-r-v-o-u-s.com/> is a really cool company that makes amazing CAM (computer-aided manufacture) jewelry, lamps, sculpture, etc. based on mathematical models of biological and physical principles. See their [projects](#) and [software](#) to see pictures and videos and to play with interactive online software (e.g., [this design tool](#)).
- Blooms and Spirals Sculptor: <https://www.youtube.com/watch?v=B5p2A5mazEs>
- Pancake printer: <http://www.eater.com/2015/3/12/8200009/pancakebot-3d-printing-pancakes-art-kickstarter>
- How *Chase Me*, a 3-D printed stop-action short film was made: <https://vimeo.com/121352977>.
- [Laika Studios](#) makes feature-length stop-action movies with 3-D printed characters: <https://www.youtube.com/channel/UCBnyjFhOn23xjsqOUW4j6OQ>.
- This company sells elaborate laser-cut press-fit models: <https://ugearsmodels.us>

A Cumulative Project

The Project: By this point in their efforts, students will have a significant skill set that they are ready to use in more open-ended ways. A *Digital Design of a Real Object* (ADDORO) project gives students the opportunity to pick an object that both interests them and that will be a challenge suitable for their current skills. Have them pick an everyday object that they can hold, measure thoroughly, and that they want to reproduce virtually. As with most Making Math projects, students are able to pick a topic or subject that will both challenge and stretch them but be within reach. In general, students have a great instinct for picking an object that will help them grow in their CAD skills but not be out of reach technically. They can look at a shape and decide if it looks too unfamiliar in terms of the 3-D surfaces that it requires. It is fine to also let students do key forms and components of a complicated object, but to leave off some lesser details. Having them keep to the main measurements remains important even if they are simplifying the object in some respects.

For teachers and students who want to see examples of how moderately complex forms can be analyzed and created with a variety of OnShape tools, the video tutorials at OnShape Learning Projects (<https://www.youtube.com/@Onshape.Learning.Projects>) are at a good level. Sample videos that may be helpful include the connecting bar ([here](#) or [here](#)), bicycle handlebars made with a sweep ([here](#)), and many other fully worked out examples.

Activity 1: Hand out the [student description](#) of the project. Before diving into OnShape, students should draw at least two orthographic views for each part and a list of variables on a copy of the [CAD Part Orthographic Diagram handout](#). The variables should have meaningful names and an initial value that matches the object's dimension. Lengths, angles, and counts should match those in the scaled orthographic drawing. The students should make numerous measurements as they create their drawings, using the scale that they set up for the drawings, and labeling lengths even if they don't merit a variable. Variables are most important for dimensions that are relevant to more than one part and for features of a shape that they might want to adjust to get variations on their object (e.g., a longer or shorter pencil, the angle at which scissors are open, etc.).

Activity 2: Spline curves. Many students' objects may involve parts that have curved outlines that are not familiar curved shapes (e.g., not arcs of circles or ellipses). To incorporate more general curves with some degree of precision (not merely sketched by eye), they should take the following steps:

1. Using their orthographic sketch, they should identify and label key points on the curve such as:

- a) the endpoints,
- b) extreme points (the tops of "mountains" or maxima and the bottoms of "valleys" or minima), and
- c) places where the curve changes from curving one way to the other (inflection points, which are momentarily flat).



2. Measure and record the horizontal and vertical distances (Δx and Δy) between each pair of consecutive points on the curve.

- Using the point tool (○) in OnShape, place as many points as were identified in steps a) through c) above in roughly the right locations relative to each other.
- Using the dimensioning tool, fix (lock in) the horizontal and vertical distances between points based on the recorded Δx and Δy values. They should be careful to not dimension the diagonal distance. If they drag straight down or straight to the side, the correct length should be established (see the dimensioning markings on the diagram below).

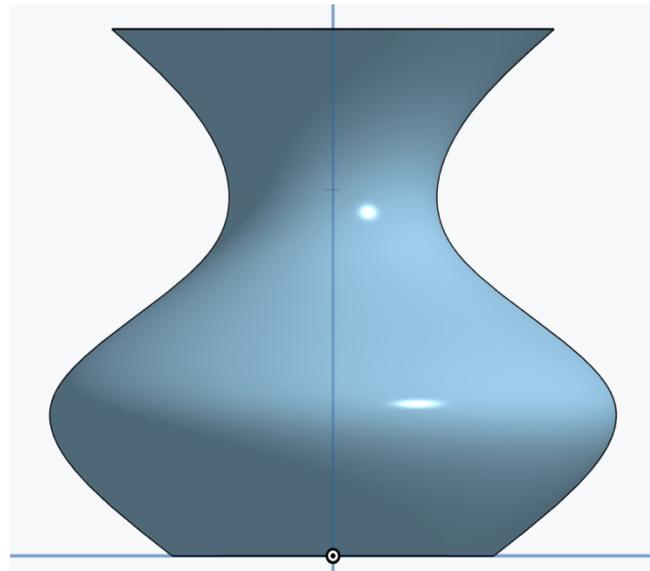
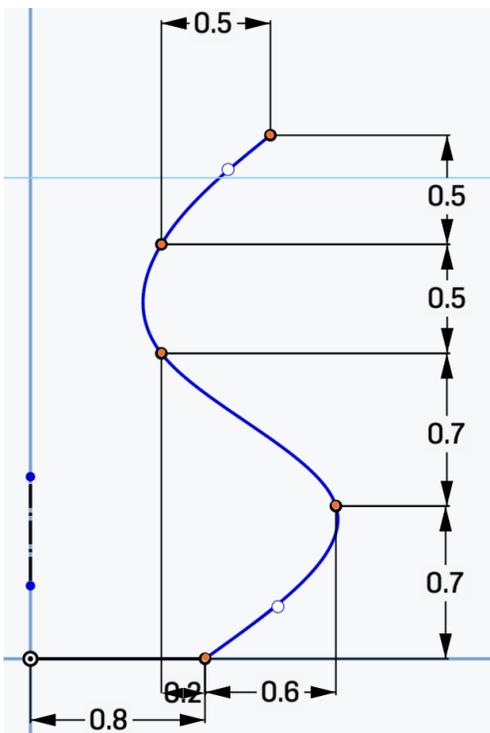
- Connect the points using OnShape's spline tool. Spline curves are piecewise polynomial functions fit to a set of points. It is important to point out that polynomials have a tendency to overshoot points. Just because we pick a point as a maximum doesn't mean the polynomial will reverse course there. For example, the orange points on the left in the diagram below were the anchor points used to generate the curve, but you can see that the curve extended beyond the two leftmost points to make a horizontal extreme point between them. If a curve needs to be controlled further, students can add additional points on the curve with the Spline control point tool.



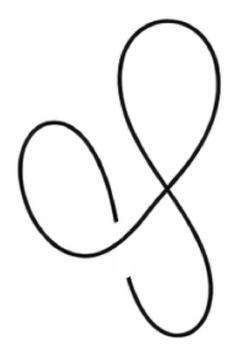
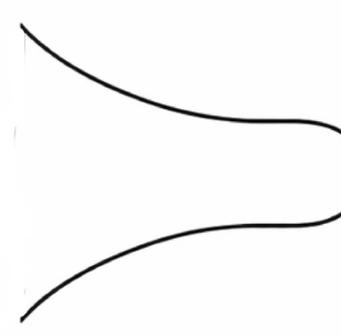
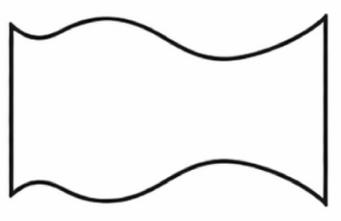
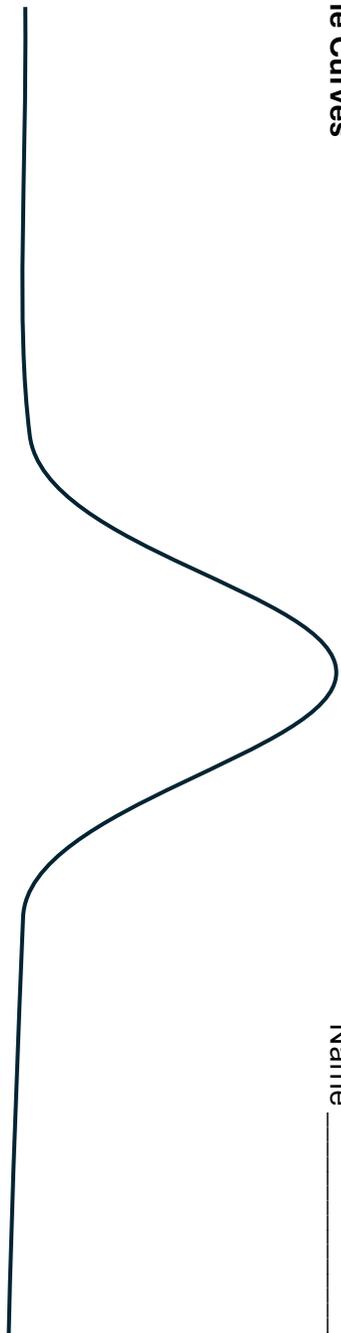
Spline



Spline control point



- Before using Splines for their own project, you can have students practice this process by picking one curve from the handout on the next page and recreating it in OnShape. Working in centimeters is a good unit at this scale.



Activity 3 – The Main Project: *A Digital Design of a Real Object Project.* This project can last anywhere from a week to several weeks depending on the complexity of the items students tackle (or are approved to tackle) and whether you want them to have time to learn new skills as the need might arise. Each project should include at least two parts, so students get to incorporate an assembly. This requirement might only necessitate a fasten mate (e.g., a pencil with an eraser) or an object with multiple parts that move relative to each other (e.g., the admittedly quite challenging antique whisk at right). One nice feature of this project is that it allows students to pick an item that is of interest to them and that is also at the level of challenge that they feel ready for. Additionally, because there are typically additional details a model could capture, students can develop a reasonable virtual version of their object and then, as time permits, keep refining it.



Here are the student steps:

Step 1) Choose an object and present a list of the main components and what they see as the main CAD step(s) to make each. Complete one or more pages of the [CAD Part Orthographic Diagram handout](#). For example, for the whisk, one might list the blue handle as a separate part (since it is made of a different material than the adjacent part) made from a revolution and the gear as an extrusion of a circular pattern.

Step 2) For the orthographic diagrams, measure the dimensions for each part in the object to the nearest millimeter (using a ruler or caliper) and present scaled orthographic projections for each part.

Step 3) Start an OnShape file and create the list of global variables that were identified on their Orthographic Diagram handouts.

Step 4) Pick the first part that they plan to draft and start the journey!

Step 5) As they complete each component from the second part onward, they should test it in an assembly at each step rather than waiting until the end. They should include the main details of their parts first and wait for less important details until they have a first full draft for all key parts of their object.

Questions to Ask at the Outset About an Object

One of the key messages in engineering is the importance of analyzing and planning in advance of design work. Here are prompts to help students make good decisions:

- What basic 2-dimensional and 3-dimensional shapes are involved in your object?

- Are there any symmetries to the object or its parts? Identifying symmetries makes it possible to avoid repetitive steps and makes shapes easier to modify efficiently and consistently. Related to symmetry is the presence of patterns.
- What is a natural view of the shape? From the top? Front? How is it oriented (what is up?).
- What bases – starting shapes for extrusions, rotations, and lofts – are there?
- What do the cross sections that are parallel to main surfaces look like?

Assessment

Teachers should strive to identify what students have mastered and what they did effectively throughout the project. Key areas of feedback in your rubric can include:

- Ability to identify and reproduce two- and three-dimensional geometric forms, symmetries, and patterns.
- Attention to precise measurement, dimensioning, and use of constraints. Do the length and angle measurements in the virtual object match the original? Are sketches fully constrained (not just drawn freehand)?
- Detail and accuracy of the initial orthographic diagrams.
- Identification and use of global variables (with clear naming habits).
- Sophistication of the chosen shape (but being clear that you should help students pick shapes that they are ready for – some students may be earlier on in their 3-D visualization skills but can still produce a sound and effective project). Are there a variety of forms within the sketches and parts?
- Effective use of a variety of OnShape tools (with the clear message that there are many correct ways to build up most shapes). Degree of independence in carrying out these steps.
- Successful assemblies (which can be the trickiest task unless more time is spent on this ahead of time – it is worth seeing this project as the time when students are getting their first consistent practice with assemblies and they may require more assistance).
- Clarity and thoroughness of the final presentation documents.

Extra Challenge

For students who finish sooner and/or who want additional skills, they can be shown how to use click on a part and control-click (or two-finger click) to bring up the menu that allows them to assign a material to each part. They can then figure out the volume, mass, and other properties of each part to determine the weight of their virtual object (how does it compare with the real object)?

Display mass and section properties



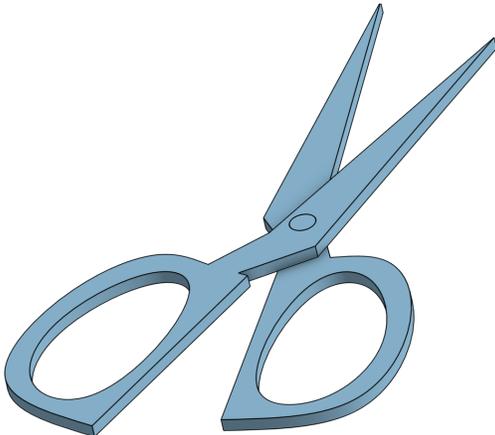
Can they come up with ways to use less material, but still have a structurally sound design? How can they use variables to explore designs and to optimize some property of their virtual object?

The following images are sample virtual objects made by students in a middle school engineering class.

Fidget Spinner



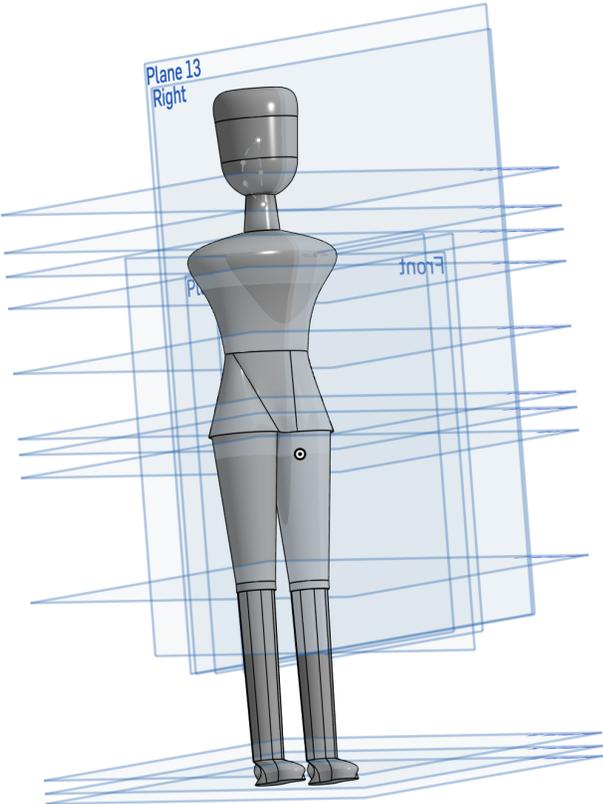
Scissors



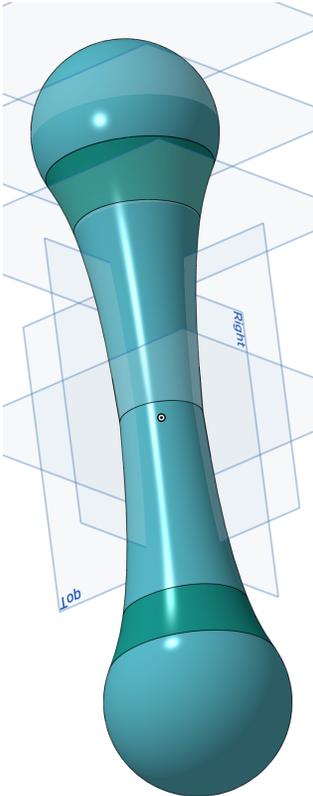
Skateboard



Mannequin



Stylish Hand Weight



Barbell



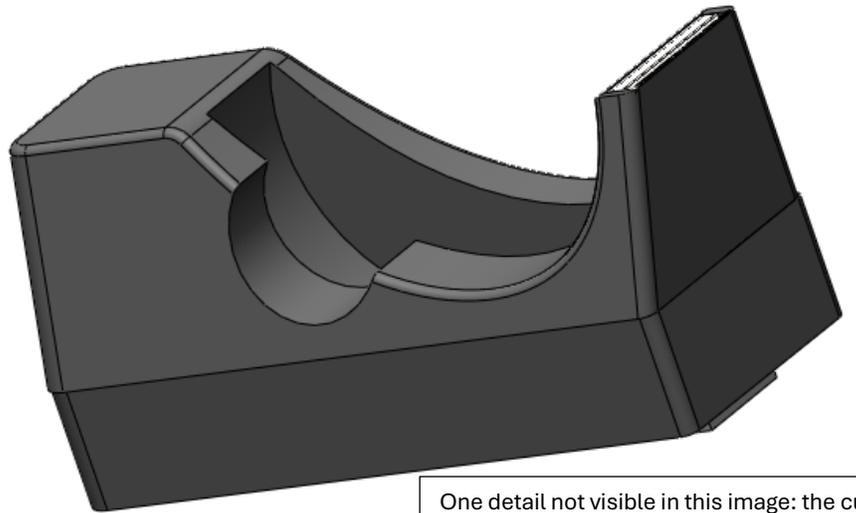
Ring



Container of Fluff



Tape Dispenser



One detail not visible in this image: the cutting edge has numerous little sharp teeth.

Mechanical Pencil

