

Afterword

The evolution of *Calculus in Context: The Five College Calculus Project* unfolded in several phases during the years from 1970 to 2026. In this Afterword, we complete the story begun in the 1995 preface.

The Five College Calculus Project

The formative phase of the Five College Calculus Project began in 1970 with the creation of Hampshire College and culminated in 1988 with a multi-year calculus curriculum development grant from the National Science Foundation. This is described in pages viii–ff of the 1995 Preface.

After the award of the NSF grant in 1988, we, in collaboration with other Five College faculty, planned and tested our new curriculum over several years. One of the provisions in the grant was for the creation of an Advisory Board for the Five College Project. The members were:

Peter Lax, Courant Institute, New York University, Chair
Solomon Garfunkel, COMAP, Inc.
John Neuberger, The University of North Texas
Barry Simon, California Institute of Technology
Gilbert Strang, Massachusetts Institute of Technology
John Truxal, State University of New York, Stony Brook

The Advisory Board provided essential support and guidance throughout the development of the text and supplementary materials. The initial 1988 grant was followed in 1991 with a multi-year dissemination grant from the NSF. See page 836 for the full list of grants that supported the Five College Calculus Project.

During this time, we presented our work at mathematics conferences and multi-day workshops around the country. Workshop participants provided valuable feedback, and some became early adopters. Barry Simon taught

the course to his Cal Tech students and completed the whole book in two semesters—possibly the only instructor ever to do so.

Publication in 1995

As noted on page xi, we were fortunate to find the publisher W. H. Freeman, whose bold and generous stance led to the publication of a calculus text that was radically different from the standard approach to calculus. However, *Calculus in Context* was not only different; it was also ahead of its time. As a result, after two press runs of 5000 each, Freeman let the book go out of print. Although disappointing, this was not the end of the story—the next phase of *Calculus in Context* was about to begin.

Online Posting in 2008 and 2020

When the book went out of print, Five Colleges, Inc., which administered the main grants supporting the Five College Calculus Project, recovered the copyright from the publisher, thus easing the way for us to publish an online version in 2008. It was posted at the website

<https://www.science.smith.edu/~callahan/intromine.html>

The 2008 version included numerous corrections but was missing the appendices and five illustrations from chapter 12 (since restored in the 2026 distribution). This website does not track downloads, so we had little idea how much continuing attention the book was receiving

Smith ScholarWorks was created in 2016 as a repository for digital scholarship and research material created by faculty, students, and staff at Smith College. The 2008 version of *Calculus in Context* was added to the repository in 2020 to ensure that the book had a stable long-term home. PDFs of the whole book and individual chapters were available. This site tracks downloads, and over 8000 downloads occurred between 2020 and 2026.

Continuing Interest After 2008

Since the posting of the 2008 version, occasionally users would write asking for the solution manual, which was not part of the distribution at the time. Others wrote with comments or questions about the material itself. Before long instructors, began to seek permission to access the L^AT_EX and PostScript source files in order to develop their own materials based on the approach taken in *Calculus in Context*.

In response to the early requests, we gave potential developers the source files on an *ad hoc* basis. One of the first recipients was Eric Slade at the University of Colorado. Starting in 2011, he adapted them to a course for life science majors, having found it easier to gain departmental acceptance for what might be called a “niche” audience. Lorenzo Sadun and William Wolesensky at the University of Texas, Austin, had a similar experience. Adapting the project’s sources, Lorenzo wrote two versions of the text, *The Six Pillars of Calculus*, one for business students and one for biology students.

The 2026 Distribution

In April 2026, Aaron Weinberg at Ithaca College requested the *Calculus in Context* source files and suggested that we consider using a Creative Commons license to make them available to all interested users. We are very grateful to Aaron for this suggestion.

After discussing various options, we decided to create a definitive, comprehensive, and final version of *Calculus in Context* that included the book, the instructor’s handbook, the solution manual, and complete source files for the book and solution manual (the source file for the handbook has been lost). This is the *2026 Distribution*, which is available under a Creative Commons license at the Smith ScholarWorks website

<https://scholarworks.smith.edu/textbooks/11/>

We thank Jessica Ryan, the Smith College scholarly communications librarian, for her assistance with Smith ScholarWorks.

Technological innovations in the last three decades make it possible to offer the book now in two forms. The first form is suitable for online use; it contains embedded hyperlinks for cross-references. The second is for printing, including printing and binding as a traditional book. For each form, there is a complete PDF file as well as all source files needed by developers. The archive also contains PDF files of individual chapters, a handbook for instructors, and a solution manual. All of this is available at the above website.

For the 2026 distribution, we changed the title to *Calculus in Context: The Five College Calculus Project* and assigned the book’s copyright to the principal authors, James Callahan and Kenneth Hoffman. The subtitle, *The Five College Calculus Project*, was added to emphasize the collaborative nature of this project and to honor Five Colleges Inc. and in particular Kenneth Hoffman and Hampshire College for the crucial contributions they made. Sadly, Ken passed away in 2024 and Hampshire closed its doors at the end of 2026.

Producing the 2026 distribution involved several different efforts. Some of the PostScript files failed to compile, but fortunately older archived versions could be found that worked properly. Then all PostScript files were converted to Encapsulated PostScript, and the `\special` commands used to include the PS files were changed to `\includegraphics` for the EPS files. This will enable developers to alter and compile the source files using *pdflatex* as well as *latex*. Five illustrations missing from the 2008 version were recreated and several other illustrations were improved. Hyperlinks for exercises were added to facilitate the use of the `hyperref` package, and numerous hyperlinked page references were added to enhance online reading. Finally, the text of the new version was corrected and updated in 2026 from the original 1995 version. The 2026 version supersedes, in every way, the 2008 version, which has now been deleted from Smith ScholarWorks.

Support

Primary funding for curriculum development and dissemination was provided by the National Science Foundation in grants DMS-14004 (1988-95) and DUE-9153301 (1991-97), awarded to Five Colleges, Inc.* Other curriculum development funding has been provided by NECUSE (New England Consortium for Undergraduate Science Education, funded by the Pew Charitable Trusts) to Smith College (1989) and Mount Holyoke College (1990). Five Colleges, Inc. also provided start-up funds.

Equipment and software for computer classrooms were funded by NSF grants in the ILI program: USE-8951485 to Smith College and DUE/EHR-9551919 to Mount Holyoke College. The Hewlett-Packard Corporation contributed equipment to Mount Holyoke and Smith Colleges, and other equipment was contributed to Mount Holyoke College by IBM and the Sloan Foundation.

Final Comment

It is our hope that the 2026 distribution will enable *Calculus in Context: The Five College Calculus Project* to continue to evolve.

August 2026

*Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation.

Appendix A

QBASIC and Graphing Calculators

A.1 QBASIC Programs

Computer code similar to BASIC appears in many places in the text. This section provides translations of these programs in QB64, a freely available version of Quick BASIC. Unlike the text, explicit graphics commands are given, and functions are defined using FUNCTION ... END FUNCTION. The QB64 programs are named as in the text, with .BAS appended.

SIR.BAS (see page 50)

```
t = 0
S = 45400
I = 2100
R = 2500
deltat = 1
PRINT t, S, I, R
FOR k = 1 TO 3
    Sprime = -.00001 * S * I
    Iprime = .00001 * S * I - I / 14
    Rprime = I / 14
    deltaS = Sprime * deltat
    deltaI = Iprime * deltat
    deltaR = Rprime * deltat
    t = t + deltat
```

```

        S = S + deltaS
        I = I + deltaI
        R = R + deltaR
        PRINT t, S, I, R
    NEXT k

```

SIRVALUE.BAS (see page 65)

```

DEFDBL A-Z
tinitial = 0
tfinal = 1
t = tinitial
S = 45400
I = 2100
R = 2500
numberofsteps = 10
deltat = (tfinal - tinitial) / numberofsteps
FOR k = 1 TO numberofsteps
    Sprime = -.00001 * S * I
    Iprime = .00001 * S * I - I / 14
    Rprime = I / 14
    deltaS = Sprime * deltat
    deltaI = Iprime * deltat
    deltaR = Rprime * deltat
    t = t + deltat
    S = S + deltaS
    I = I + deltaI
    R = R + deltaR
NEXT k
PRINT t, S, I, R

```

SIRPLOT.BAS (see page 69)

```

DEFDBL A-Z
SCREEN 12
WINDOW (0, 40000)-(3, 45400)
tinitial = 0
tfinal = 3
t = tinitial
S = 45400
I = 2100
R = 2500
numberofsteps = 30
deltat = (tfinal - tinitial) / numberofsteps
FOR k = 1 TO numberofsteps

```

```

    Sprime = -.00001 * S * I
    Iprime = .00001 * S * I - I / 14
    Rprime = I / 14
    deltaS = Sprime * deltat
    deltaI = Iprime * deltat
    deltaR = Rprime * deltat
    LINE (t, S)-(t + deltat, S + deltaS)
    t = t + deltat
    S = S + deltaS
    I = I + deltaI
    R = R + deltaR
NEXT k

```

DOWHILE.BAS (see SIRPLOT with a DO-WHILE loop, page 77)

```

DEFDBL A-Z
SCREEN 12
WINDOW (0, 0)-(20, 47000)
tinitial = 0
t = tinitial
S = 45400
I = 2100
R = 2500
Iprime = .00001 * S * I - I / 14
deltat = .01
DO WHILE Iprime > 0
    Sprime = -.00001 * S * I
    Iprime = .00001 * S * I - I / 14
    Rprime = I / 14
    deltaS = Sprime * deltat
    deltaI = Iprime * deltat
    deltaR = Rprime * deltat
    LINE (t, S)-(t + deltat, S + deltaS)
    t = t + deltat
    S = S + deltaS
    I = I + deltaI
    R = R + deltaR
LOOP
PRINT t

```

SEQUENCE.BAS (see page 83)

```

DEFDBL A-Z
SCREEN 9
COLOR 8, 7
WINDOW (0, 0)-(75, 1300)

```

```

FOR j = 1 TO 14
    tinitial = 0
    tfinal = 75
    t = tinitial
    y = 100
    numberofsteps = 2 ^ (j - 1)
    deltat = (tfinal - tinitial) / numberofsteps
    FOR k = 1 TO numberofsteps
        yprime = .1 * y * (1 - y / 1000)
        deltay = yprime * deltat
        LINE (t, y)-(t + deltat, y + deltay), j
        t = t + deltat
        y = y + deltay
    NEXT k
NEXT j

```

LENGTH.BAS (see page 91)

```

DEFDBL A-Z
xinitial = 0
xfinal = 1
n = 2
deltax = (xfinal - xinitial) / n
FOR k = 1 TO n
    xl = xinitial + (k - 1) * deltax
    xr = xinitial + k * deltax
    yl = fnf(xl)
    yr = fnf(xr)
    segment = SQR((xr - xl) ^ 2 + (yl - yr) ^ 2)
    total = total + segment
    PRINT k, segment
NEXT k
PRINT n, total
FUNCTION fnf (x)
    fnf = x ^ 2
END FUNCTION

```

BABYLON.BAS (see page 94)

```

DEFDBL A-Z
a = 5
x = 2
n = 6
FOR k = 1 TO n
    x = (x + a / x) / 2
    PRINT x
NEXT k

```

TABLE.BAS (see page 266)

```

DEFDBL A-Z
tinitial = 0
tfinal = 4
t = tinitial
numberofsteps = 2 ^ 3
deltat = (tfinal - tinitial) / numberofsteps
accumulation = 0
FOR k = 1 TO numberofsteps
    deltay = fnf(t) * deltat
    accumulation = accumulation + deltay
    t = t + deltat
    PRINT deltat, accumulation, t
NEXT k
FUNCTION fnf (t)
    fnf = COS(t ^ 2)
END FUNCTION

```

PLOT.BAS (see page 268)

```

DEFDBL A-Z
SCREEN 12
WINDOW (0, 0)-(4, 2)
tinitial = 0
tfinal = 4
t = tinitial
accumulation = 0
numberofsteps = 400
deltat = (tfinal - tinitial) / numberofsteps
FOR k = 1 TO numberofsteps
    deltay = fnf(t) * deltat
    LINE (t, accumulation)-(t + deltat, accumulation + deltay)
    accumulation = accumulation + deltay
    t = t + deltat
NEXT k
FUNCTION fnf (t)
    fnf = COS(t ^ 2)
END FUNCTION

```

NEWTON.BAS (see page 321)

```

DEFDBL A-Z
start = -1
numberofsteps = 8
x = start

```

```

FOR n = 0 TO numberofsteps
    PRINT n, x
    g = 4 * x ^ 3 + 3 * x ^ 2 + 2 * x + 1
    gprime = 12 * x ^ 2 + 6 * x + 2
    x = x - g / gprime
NEXT n

```

RIEMANN.BAS (see page 360)

```

DEFDBL A-Z
a = 1
b = 3
numberofsteps = 4
deltax = (b - a) / numberofsteps
x = a
accumulation = 0
FOR k = 1 TO numberofsteps
    deltaS = fnf(x) * deltax
    accumulation = accumulation + deltaS
    x = x + deltax
    PRINT deltaS, accumulation
NEXT k
FUNCTION fnf (x)
    fnf = SQR(1 + x ^ 3)
END FUNCTION

```

TAYLOR.BAS (see page 600)

```

DEFDBL A-Z
SCREEN 12
WINDOW (0, -1.5)-(3.14, 1.5)
FOR x = 0 TO 3.14 STEP .01
    LINE (x, fnpoly(x))-(x + .01, fnpoly(x + .01))
NEXT x
FUNCTION fnfact (m)
    P = 1
    FOR r = 2 TO m
        P = P * r
    NEXT r
    fnfact = P
END FUNCTION
FUNCTION fnpoly (x)
    Sum = x
    Sign = -1
    FOR k = 3 TO 17 STEP 2
        Sum = Sum + Sign * x ^ k / fnfact(k)
    NEXT k
END FUNCTION

```

```

                Sign = (-1) * Sign
            NEXT k
            fnpoly = Sum
END FUNCTION

```

SIRSERIES.BAS (see page 636)

```

DEFDBL A-Z
DIM s(50), i(50)
a = .00001q
b = 1 / 14
s(0) = 45400
i(0) = 2100
FOR k = 1 TO 50
    sum = 0
    FOR j = 0 TO k - 1
        sum = sum + s(j) * i(k - j - 1)
    NEXT j
    s(k) = -a * sum / k
    i(k) = -s(k) - b * i(k - 1) / k
NEXT k

```

HARMONIC.BAS (see page 662)

```

n = 100
sum = 0
FOR i = 1 TO n
    sum = sum + 1/i
NEXT i
PRINT n, sum

```

DETECTOR.BAS (page 803)

```

DEFDBL A-Z
SCREEN 12
WINDOW (0, -1)-(3, 2)
startomega = 0
endomega = 3
omegasteps = 2 ^ 10
deltaomega = (endomega - startomega) / omeagasteps
twopi = 8 * ATN(1)
a = 0
b = 10
numberofsteps = 2 ^ 7
deltat = (b - a) / numberofsteps
omega = startomega

```

```

oldomega = omega
oldaccum = 0
FOR j = 1 TO omegasteps
    t = a + deltata / 2
    accum = 0
    FOR k = 1 TO numberofsteps
        deltaS = (fnf(t) * SIN(twopi * omega * t) * deltata) / (b - a)
        accum = accum + deltaS
        t = t + deltata
    NEXT k
    omega = omega + deltaomega
    LINE (oldomega, oldaccum)-(omega, accum)
    oldomega = omega
    oldaccum = accum
NEXT j
FUNCTION fnf (t)
    fnf = 3 * SIN(5 * t)
END FUNCTION

```

POWER.BAS (see page 810)

```

DEFDBL A-Z
SCREEN 9
COLOR 8, 7
WINDOW (0, -.5)-(3, 4)
startomega = 0
endomega = 3
omegasteps = 2 ^ 9
deltaomega = (endomega - startomega) / omegasteps
pi = 4 * ATN(1)
twopi = 2 * pi
a = 0
b = 10
numberofsteps = 2 ^ 6
deltata = (b - a) / numberofsteps
omega = startomega
oldomega = omega
oldpower = 0
FOR j = 1 TO omegasteps
    t = a + deltata / 2
    accumS = 0
    accumC = 0
    power = 0
    FOR k = 1 TO numberofsteps
        deltaS = 2 * fnf(t) * SIN(twopi * omega * t) * deltata / (b - a)
        accumS = accumS + deltaS

```

```

        deltaC = 2 * fnf(t) * COS(twopi * omega * t) * deltat / (b - a)
        accumC = accumC + deltaC
        t = t + deltat
    NEXT k
    power = SQR(accumS ^ 2 + accumC ^ 2)
    omega = omega + deltaomega
    LINE (oldomega, oldpower)-(omega, power)
    oldomega = omega
    oldpower = power
NEXT j
LINE (7 / twopi, 0)-(7 / twopi, 3), 6    'this marks the location of the
                                         'peak in the power spectrum

FUNCTION fnf (t)
    fnf = 3 * SIN(7 * t - pi / 3)
END FUNCTION

```

We note that the program “modified SIRPLOT” on page 80 and the code fragment “Extension to SIRSERIES” on page 638 are not included here.

A.2 Graphing Calculators

The remainder of this appendix contains translations of the most of the QBASIC programs that appear in the previous section into programs for various graphing calculators.

Each calculator program has the same name as the corresponding QBASIC program. Wherever possible, the calculator’s code has been written to mirror the computer’s, and we note points where the two differ. In particular, a variable name may reduce to just a single letter in the calculator. Comments in the text concerning aspects of a computer program generally apply as well to the corresponding calculator program.

Two of the QBASIC programs, DETECTOR and POWER, make unreasonable demands on the graphing calculators featured here. For example, POWER takes more than 45 minutes to run on a TI-82. For this reason, we have not included translations of these programs (except for the TI-82). However, the programs themselves contain no new elements; you should be able to translate them for any of the graphing calculators if you wish.

We assume that you are already familiar with the general aspects of programming a calculator and that you know which keystrokes on your own calculator will produce these programs. This appendix covers five popular kinds of graphing calculators that were available in the early 1990s.

A.3 Texas Instruments TI-81

SIR

The program pauses each time it displays t , S , I , and R . Press ENTER to continue.

```
PROGRAM:SIR
:0→T
:45400→S
:2100→I
:2500→R
:Disp "T,S,I,R"
:Disp T
:Disp S
:Disp I
:Disp R
:1→D
:1→K
:Lbl 1
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:Pause
:Disp T
:Disp S
:Disp I
:Disp R
:IS>(K,3)
:Goto 1
```

SIRVALUE

```
PROGRAM:SIRVALUE
:0→F
:1→L
:F→T
:45400→S
:2100→I
:2500→R
:10→N
:(L-F)/N→D
:1→K
:Lbl 1
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:IS>(K,N)
:Goto 1
:Disp T
:Disp S
:Disp I
:Disp R
```

Note: Be sure to distinguish between “0” (*zero*) and “O” (*Oh*).

SIRPLOT

```

PROGRAM:SIRPLOT
:0→F
:3→L
:F→T
:45400→S
:2100→I
:2500→R
:30→N
:All-Off
:ClrDraw
:F→Xmin
:L→Xmax
:1→Xscl
:0→Ymin
:S→Ymax
:10000→Yscl
:(L-F)/N→D
:1→K
:Lbl 1
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:Line(T,S,T+D,S+U)
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:IS>(K,N)
:Goto 1

```

DOWHILE

```

PROGRAM:DOWHILE
:0→F
:F→T
:45400→S
:2100→I
:2500→R
: .00001*S*I-I/14→P
:All-Off
:ClrDraw
:F→Xmin
:20→Xmax
:5→Xscl
:0→Ymin
:S→Ymax
:10000→Yscl
: .01→D
:Lbl 1
:If P≤0
:Goto 2
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:Line(T,S,T+D,S+U)
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:B→P
:Goto 1
:Lbl 2
:Disp T

```

SEQUENCE

Because of the speed and screen size of the calculator, this program has been limited to plotting the first 8 approximations to $y(t)$.

PROGRAM: SEQUENCE

```
:All-Off
:ClrDraw
:0→Xmin
:75→Xmax
:10→Xscl
:0→Ymin
:1500→Ymax
:500→Yscl
:1→J
:Lbl 1
:0→F
:75→L
:F→T
:100→V
:2^(J-1)→N
:(L-F)/N→D
:1→K
:Lbl 2
:.1*V*(1-V/1000)→P
:P*D→C
:Line(T,V,T+D,V+C)
:T+D→T
:V+C→V
:IS>(K,N)
:Goto 2
:IS>(J,8)
:Goto 1
```

LENGTH

The program pauses after displaying the length of each segment. Press to continue.

PROGRAM: LENGTH

```
: "X2"→Y1
:0→F
:1→L
:2→N
:(L-F)/N→D
:0→T
:1→K
:Lbl 1
:F+(K-1)*D→A
:A→X
:Y1→Y
:F+K*D→B
:B→X
:√((B-A)2+(Y1-Y)2)→S
:T+S→T
:Disp K
:Disp S
:Pause
:IS>(K,N)
:Goto 1
:Disp "TOTAL"
:Disp N
:Disp T
```

TABLE

The program pauses after displaying the current value of Δy , the accumulated Δy , and the ending t . Press **ENTER** to continue.

```
PROGRAM:TABLE
:"cos X^2"→Y1
:Rad
:0→F
:4→L
:2^3→N
:(L-F)/N→D
:F→T
:0→A
:1→K
:Lbl 1
:Y1*D→Y
:A+Y→A
:T+D→T
:Disp Y
:Disp A
:Disp T
:Pause
:IS>(K,N)
:Goto 1
:Disp "ACCUMULATION"
:Disp A
```

PLOT

```
PROGRAM:PLOT
:ClrDraw
:0→Xmin
:4→Xmax
:1→Xscl
:0→Ymin
:1→Ymax
:1→Yscl
:"cos X^2"→Y1
:Rad
:All-Off
:0→F
:4→L
:400→N
:(L-F)/N→D
:F→T
:0→A
:1→K
:Lbl 1
:T→X
:Y1*D→V
:Line(T,A,T+D,A+V)
:A+V→A
:T+D→T
:IS>(K,N)
:Goto 1
```

NEWTON

The program pauses after displaying each successive approximation. Press to continue.

```
PROGRAM:NEWTON
:-1→S
:8→N
:S→X
:0→J
:Lbl 1
:Disp J
:Disp X
:4X^3+3X^2+2X+1→G
:12X^2+6X+2→P
:X-G/P→X
:Pause
:IS>(J,N)
:Goto 1
```

RIEMANN

The program pauses after displaying each successive accumulation. Press to continue.

```
PROGRAM:RIEMANN
:"√(1+X^3)"→Y1
:1→A
:3→B
:4→N
:(B-A)/N→D
:A→X
:0→C
:1→K
:Lbl 1
:Y1*D→S
:C+S→C
:X+D→X
:Disp S
:Disp C
:Pause
:IS>(K,N)
:Goto 1
```

BABYLON

```
PROGRAM:BABYLON
:5→A
:2→X
:6→N
:1→K
:Lbl 1
:(X+A/X)/2→X
:Disp X
:IS>(K,N)
:Goto 1
```

HARMONIC

```
PROGRAM:HARMONIC
:100→N
:0→S
:1→I
:Lbl 1
:S+1/I→S
:IS>(I,N)
:Goto 1
:Disp N
:Disp S
```

TAYLOR

There are *two* programs here. The first, PTAYLOR, evaluates a 17th degree Taylor polynomial at the current value of x . It is a subroutine called by the main program, TAYLOR, which refers to it as PrgmZ. Replace PrgmZ by the name your calculator uses to refer to PTAYLOR.

PROGRAM:PTAYLOR

```
:X→S
:-1→U
:3→K
:Lbl 2
:S+U*X^K/K!→S
:-1U→U
:K+2→K
:If K≤17
:Goto 2
:End
```

PROGRAM:TAYLOR

```
:All-Off
:ClrDraw
:0→Xmin
:3.14→Xmax
:1→Xscl
:-1.5→Ymin
:1.5→Ymax
:1→Yscl
:0→X
:PrgmZ
:S→T
:.01→J
:Lbl 1
:J→X
:PrgmZ
:Line(J-.01,T,J,S)
:S→T
:J+.01→J
:If J≤3.14
:Goto 1
:Stop
```

SIRSERIES

This program clears the statistical registers, so take care to back them up if they contain important data. The first 50 coefficients in the power series for $S(t)$ (that is, the coefficients of $t^0, \dots, t^{49}$) are stored in {x} and the coefficients for $I(t)$ are stored in {y}. Note that the elements in these TI-81 lists are numbered from 1 to 50, so the coefficient of t^{49} in the power series for $S(t)$ would be {x}(50).

PROGRAM:SIRSERIE

```
:.00001→A
:1/14→B
:ClrStat
:45400→{x}(1)
:2100→{y}(1)
:1→K
:Lbl 1
:0→M
:0→J
:Lbl 2
:M+{x}(J+1)*{y}(K-J)→M
:IS>(J,K-1)
:Goto 2
:-A*M/K→{x}(K+1)
:-{x}(K+1)-B*{y}(K)/K→{y}(K+1)
:IS>(K,49)
:Goto 1
```

A.4 Texas Instruments TI-82

SIR

The program pauses each time it displays t , S , I , and R . Press ENTER to continue.

```
PROGRAM:SIR
:0→T
:45400→S
:2100→I
:2500→R
:Disp "T,S,I,R",T,S,I,R
:1→D
:For(K,1,3)
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:Pause
:Disp T,S,I,R
:End
```

SIRVALUE

```
PROGRAM:SIRVALUE
:0→F
:1→L
:F→T
:45400→S
:2100→I
:2500→R
:10→N
:(L-F)/N→D
:For(K,1,N)
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:End
:Disp T,S,I,R
```

Note: Be sure to distinguish between “0” (*zero*) and “O” (*Oh*).

SIRPLOT

```

PROGRAM:SIRPLOT
:0→F
:3→L
:F→T
:45400→S
:2100→I
:2500→R
:30→N
:FnOff
:ClrDraw
:F→Xmin
:L→Xmax
:1→Xscl
:0→Ymin
:S→Ymax
:10000→Yscl
:(L-F)/N→D
:For(K,1,N)
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:Line(T,S,T+D,S+U)
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:End

```

DOWHILE

```

PROGRAM:DOWHILE
:0→F
:F→T
:45400→S
:2100→I
:2500→R
: .00001*S*I-I/14→P
:FnOff
:ClrDraw
:F→Xmin
:20→Xmax
:5→Xscl
:0→Ymin
:S→Ymax
:10000→Yscl
: .01→D
:While P>0
: -.00001*S*I→A
: .00001*S*I-I/14→B
:I/14→C
:A*D→U
:B*D→V
:C*D→W
:Line(T,S,T+D,S+U)
:T+D→T
:S+U→S
:I+V→I
:R+W→R
:B→P
:End
:Disp T

```

SEQUENCE

Because of the speed and screen size of the calculator, this program has been limited to plotting the first 8 approximations to $y(t)$.

```
PROGRAM:SEQUENCE
:FnOff
:ClrDraw
:0→Xmin
:75→Xmax
:10→Xscl
:0→Ymin
:1500→Ymax
:500→Yscl
:For(J,1,8)
:0→F
:75→L
:F→T
:100→V
:2^(J-1)→N
:(L-F)/N→D
:For(K,1,N)
:.1*V*(1-V/1000)→P
:P*D→C
:Line(T,V,T+D,V+C)
:T+D→T
:V+C→V
:End
:End
```

LENGTH

The program pauses after displaying the length of each segment. Press to continue.

```
PROGRAM:LENGTH
:"X^2"→Y1
:0→F
:1→L
:2→N
:(L-F)/N→D
:0→T
:For(K,1,N)
:F+(K-1)*D→A
:F+K*D→B
:√((B-A)^2+(Y1(B)-Y1(A))^2)→S
:T+S→T
:Disp K,S
:Pause
:End
:Disp "steps",N
:Disp "length",T
```

TABLE

This program deletes the lists L_1 , L_2 , and L_3 , so take care to back them up if they contain important data. The current value of Δy is stored in L_1 ; the accumulated Δy is stored in L_2 ; and the ending t is stored in L_3 .

PROGRAM:TABLE

```
: "cos X^2" → Y1
: Radian
: ClrList L1, L2, L3
: 0 → F
: 4 → L
: 2^3 → N
: (L-F)/N → D
: F → T
: 0 → A
: For(K, 1, N)
: Y1(T)*D → Y
: A+Y → A
: T+D → T
: Y → L1(K)
: A → L2(K)
: T → L3(K)
: End
: Disp "ACCUMULATION", L2(N)
```

PLOT

PROGRAM:PLOT

```
: ClrDraw
: 0 → Xmin
: 4 → Xmax
: 1 → Xscl
: 0 → Ymin
: 1 → Ymax
: 1 → Yscl
: "cos X^2" → Y1
: FnOff
: Radian
: 0 → F
: 4 → L
: 400 → N
: (L-F)/N → D
: F → T
: 0 → A
: For(K, 1, N)
: Y1(T)*D → V
: Line(T, A, T+D, A+V)
: A+V → A
: T+D → T
: End
```

NEWTON

The program pauses after displaying each successive approximation. Press **ENTER** to continue.

```
PROGRAM:NEWTON
:-1→S
:8→N
:S→X
:For(J,0,N)
:Disp J,X
:4X^3+3X^2+2X+1→G
:12X^2+6X+2→P
:X-G/P→X
:Pause
:End
```

RIEMANN

This program deletes the lists L_1 and L_2 , so take care to back them up if they contain important data. Incremental values are stored in L_1 and the accumulation is stored in L_2 .

```
PROGRAM:RIEMANN
:"√(1+X^3)"→Y1
:ClrList L1,L2
:1→A
:3→B
:4→N
:(B-A)/N→D
:A→X
:0→C
:For(K,1,N)
:Y1(X)*D→S
:C+S→C
:X+D→X
:S→L1(K)
:C→L2(K)
:End
:Disp L2(N)
```

BABYLON

```
PROGRAM:BABYLON
:5→A
:2→X
:6→N
:For(K,1,N)
:(X+A/X)/2→X
:Disp X
:End
```

HARMONIC

```
PROGRAM:HARMONIC
:100→N
:0→S
:For(I,1,N)
:S+1/I→S
:End
:Disp N,S
```

TAYLOR

There are *two* programs here. The first, called PTAYLOR, evaluates a 17th degree Taylor polynomial for the current value of x . It is a sub-routine called by TAYLOR, the main program, when it is needed.

```

PROGRAM:PTAYLOR
:X→S
:-1→U
:For(K,3,17,2)
:S+U*X^K/K!→S
:-1U→U
:End
PROGRAM:TAYLOR
:FnOff
:ClrDraw
:0→Xmin
:3.14→Xmax
:1→Xscl
:-1.5→Ymin
:1.5→Ymax
:1→Yscl
:0→X
:prgmPTAYLOR
:S→T
:For(J,.01,3.14,.01)
:J→X
:prgmPTAYLOR
:Line(J-.01,T,J,S)
:S→T
:End

```

SIRSERIES

This program deletes the lists L_1 and L_2 , so take care to back them up if they contain important data. The first 51 coefficients in the power series for $S(t)$ (that is, the coefficients of $t^0, \dots, t^{50}$) are stored in L_1 and the coefficients for $I(t)$ are stored in L_2 . Note that the elements in these TI-82 lists are numbered from 1 to 51, so the coefficient of t^{50} in the power series for $S(t)$ would be $L_1(51)$.

```

PROGRAM:SIRSERIE
:ClrList L1,L2
:.00001→A
:1/14→B
:45400→L1(1)
:2100→L2(1)
:For(K,1,50)
:0→S
:For(J,0,K-1)
:S+L1(J+1)*L2(K-J)→S
:End
:-A*S/K→L1(K+1)
:-L1(K+1)-B*L2(K)/K→L2(K+1)
:End

```

DETECTOR

```

PROGRAM:DETECTOR
:ClrDraw
:0→F
:3→L
:F→Xmin
:L→Xmax
:1/(2π)→Xscl
:-.2→Ymin
:1.4→Ymax
:.5→Yscl
:2^6→N
:(L-F)/N→D
:"3sin 5X"→Y1
:Radian
:FnOff
:0→A
:10→B
:2^5→S
:(B-A)/S→T
:F→M
:M→G
:0→U
:For(J,1,N)
:A+T/2→X
:0→C
:For(K,1,S)
:Y1(X)*sin (2π*M*X)*D/(B-A)→E
:C+E→C
:X+T→X
:End
:M+D→M
:Line(G,U,M,C)
:M→G
:C→U
:End

```

POWER

```

PROGRAM:POWER
:ClrDraw
:0→F
:3→L
:F→Xmin
:L→Xmax
:1/(2π)→Xscl
:-.2→Ymin
:3.5→Ymax
:1→Yscl
:2^7→N
:(L-F)/N→D
:"3sin (7X-π/3)"→Y1
:Radian
:FnOff
:0→A
:10→B
:2^6→S
:(B-A)/S→T
:F→M
:M→G
:0→U
:For(J,1,N)
:A+T/2→X
:0→Q
:0→R
:0→P
:For(K,1,S)
:2*Y1(X)*sin (2π*M*X)*T/(B-A)→V
:Q+V→Q
:2*Y1(X)*cos (2π*M*X)*T/(B-A)→W
:R+W→R
:X+T→X
:End
:√(Q2+R2)→P
:M+D→M
:Line(G,U,M,P)
:M→G
:P→U
:End

```

A.5 Texas Instruments TI-85

SIR

The program pauses each time it displays t , S , I , and R . Press to continue.

```
PROGRAM:SIR
:0→t
:45400→S
:2100→I
:2500→R
:1→deltat
:Disp t,S,I,R
:For(K,1,3)
: -.00001*S*I→Sprime
: .00001*S*I-I/14→Iprime
:I/14→Rprime
:Sprime*deltat→deltaS
:Iprime*deltat→deltaI
:Rprime*deltat→deltaR
:t+deltat→t
:S+deltaS→S
:I+deltaI→I
:R+deltaR→R
:Pause
:Disp t,S,I,R
:End
```

SIRVALUE

```
PROGRAM:SIRVALUE
:0→tinitial
:1→tfinal
:tinitial→t
:45400→S
:2100→I
:2500→R
:10→numsteps
:(tfinal-tinitial)/numsteps
  →deltat
:For(K,1,numsteps)
: -.00001*S*I→Sprime
: .00001*S*I-I/14→Iprime
:I/14→Rprime
:Sprime*deltat→deltaS
:Iprime*deltat→deltaI
:Rprime*deltat→deltaR
:t+deltat→t
:S+deltaS→S
:I+deltaI→I
:R+deltaR→R
:End
:Disp t,S,I,R
```

Note: Be sure to distinguish between “0” (*zero*) and “0” (*Oh*).

SIRPLOT

```

PROGRAM:SIRPLOT
:0→tinitial
:3→tfinal
:tinitial→t
:45400→S
:2100→I
:2500→R
:30→numsteps
:FnOff
:ClDrw
:tinitial→xMin
:tfinal→xMax
:1→xScl
:0→yMin
:S→yMax
:10000→yScl
:(tfinal-tinitial)/numsteps
  →deltat
:For(K,1,numsteps)
:-.00001*S*I→Sprime
:.00001*S*I-I/14→Iprime
:I/14→Rprime
:Sprime*deltat→deltaS
:Iprime*deltat→deltaI
:Rprime*deltat→deltaR
:Line(t,S,t+deltat,S+deltaS)
:t+deltat→t
:S+deltaS→S
:I+deltaI→I
:R+deltaR→R
:End

```

DOWHILE

```

PROGRAM:DOWHILE
:0→tinitial
:tinitial→t
:45400→S
:2100→I
:2500→R
:FnOff
:ClDrw
:tinitial→xMin
:20→xMax
:5→xScl
:0→yMin
:S→yMax
:10000→yScl
:.00001*S*I-I/14→Iprime
:.01→deltat
:While Iprime>0
:-.00001*S*I→Sprime
:.00001*S*I-I/14→Iprime
:I/14→Rprime
:Sprime*deltat→deltaS
:Iprime*deltat→deltaI
:Rprime*deltat→deltaR
:Line(t,S,t+deltat,S+deltaS)
:t+deltat→t
:S+deltaS→S
:I+deltaI→I
:R+deltaR→R
:End
:Disp t

```

SEQUENCE

Because of the speed and screen size of the calculator, this program has been limited to plotting the first 8 approximations to $y(t)$.

```
PROGRAM:SEQUENCE
:FnOff
:C1Drw
:0→xMin
:75→xMax
:10→xScl
:0→yMin
:1500→yMax
:500→yScl
:For(J,1,8)
:0→tinitial
:75→tfinal
:tinitial→t
:100→y
:2^(J-1)→numsteps
:(tfinal-tinitial)/numsteps
  →deltat
:For(K,1,numsteps)
:.1*y*(1-y/1000)→yprime
:yprime*deltat→deltay
:Line(t,y,t+deltat,y+deltay)
:t+deltat→t
:y+deltay→y
:End
:End
```

LENGTH

The program pauses after displaying the length of each segment. Press **ENTER** to continue.

```
PROGRAM:LENGTH
:y1=x^2
:0→xinitial
:1→xfinal
:2→N
:(xfinal-xinitial)/N→deltax
:0→total
:For(K,1,N)
:xinitial+(K-1)*deltax→XL
:xinitial+K*deltax→XR
:XL→x
:y→YL
:XR→x
:y→YR
:√((XR-XL)^2+(YR-YL)^2)→segment
:total+segment→total
:Disp K,segment
:Pause
:End
:Disp "steps",N,"total",total
```

TABLE

The program pauses after displaying the current value of Δy , the accumulated Δy , and the ending t . Press ENTER to continue.

```
PROGRAM:TABLE
:y1=cos x^2
:Radian
:0→tinitial
:4→tfinal
:2^3→numsteps
:(tfinal-tinitial)/numsteps
  →deltat
:tinitial→t
:0→accum
:For(K,1,numsteps)
:t→x
:y1*deltat→deltay
:accum+deltay→accum
:t+deltat→t
:Disp deltay,accum,t
:Pause
:End
:Disp "accum",accum
```

PLOT

```
PROGRAM:PLOT
:C1Drw
:0→xMin
:4→xMax
:1→xScl
:0→yMin
:1→yMax
:1→yScl
:y1=cos x^2
:FnOff
:Radian
:0→tinitial
:4→tfinal
:400→numsteps
:(tfinal-tinitial)/numsteps
  →deltat
:tinitial→t
:0→accum
:For(K,1,numsteps)
:t→x
:y1*deltat→deltay
:Line(t,accum,t+deltat,
      accum+deltay)
:accum+deltay→accum
:t+deltat→t
:End
```

NEWTON

The program pauses after displaying each successive approximation. Press **ENTER** to continue.

```
PROGRAM:NEWTON
:-1→start
:8→numsteps
:start→x
:For(N,0,numsteps)
:Disp N,x
:G=4x^3+3x^2+2x+1
:Gprime=12x^2+6x+2
:x-G/Gprime→x
:Pause
:End
```

BABYLON

```
PROGRAM:BABYLON
:5→A
:2→x
:6→N
:For(K,1,N)
:(x+A/x)/2→x
:Disp x
:End
```

RIEMANN

The program pauses after displaying each successive accumulation. Press **ENTER** to continue.

```
PROGRAM:RIEMANN
:y1=√(1+x^3)
:1→A
:3→B
:4→numsteps
:(B-A)/numsteps→deltax
:A→x
:0→accum
:For(K,1,numsteps)
:y1*deltax→deltaS
:accum+deltaS→accum
:x+deltax→x
:Disp deltaS,accum
:Pause
:End
```

HARMONIC

```
PROGRAM:HARMONIC
:100→N
:0→sm
:For(I,1,N)
:sm+1/I→sm
:End
:Disp N,sm
```

TAYLOR

There are *two* programs here. The first, called PTAYLOR, evaluates a 17th degree Taylor polynomial for the current value of x . It is a subroutine called by TAYLOR, the main program, when it is needed.

```

PROGRAM:PTAYLOR
:x→sm
:-1→sgn
:For(K,3,17,2)
:sm+sgn*x^K/K!→sm
:-1*sgn→sgn
:End
:sm→Taylor

PROGRAM:TAYLOR
:FnOff
:ClDrw
:0→xMin
:3.14→xMax
:1→xSc1
:-1.5→yMin
:1.5→yMax
:1→ySc1
:0→x
:PTAYLOR
:Taylor→T
:For(x,.01,3.14,.01)
:PTAYLOR
:Line(x-.01,T,x,Taylor)
:Taylor→T
:End

```

SIRSERIES

This program deletes the contents of lists S and I , so take care to back them up if they contain important data. Note that the elements in these TI-85 lists are numbered from 1 to 51, so the coefficient of t^{50} in the power series for $S(t)$ would be S(51).

```

PROGRAM:SIRSERIE
:51→dimL S
:51→dimL I
:.00001→A
:1/14→B
:45400→S(1)
:2100→I(1)
:For(K,1,50)
:0→sm
:For(J,0,K-1)
:sm+S(J+1)*I(K-J)→sm
:End
:-A*sm/K→S(K+1)
:-S(K+1)-B*I(K)/K→I(K+1)
:End

```

A.6 Casio *fx-7700G/fx-9700GE*

SIR

The program pauses each time it displays t , S , I , and R . Press ENTER to continue.

```
SIR
"T, S, I, R"
0→T:T▲
45400→S:S▲
2100→I:I▲
2500→R:R▲
1→D
1→K
Lb1 1
-.00001SI→A
.00001SI-I÷14→B
I÷14→C
AD→U
BD→V
CD→W
T+D→T:T▲
S+U→S:S▲
I+V→I:I▲
R+W→R:R▲
Isz K:K≤3⇒Goto 1
```

SIRVALUE

```
SIRVALUE
0→F
1→L
F→T
45400→S
2100→I
2500→R
10→N
(L-F)÷N→D
1→K
Lb1 1
-.00001SI→A
.00001SI-I÷14→B
I÷14→C
AD→U
BD→V
CD→W
T+D→T
S+U→S
I+V→I
R+W→R
Isz K:K≤N⇒Goto 1
T▲
S▲
I▲
R
```

Note: Be sure to distinguish between “0” (*zero*) and “0” (*Oh*).

SIRPLOT

```

SIRPLOT
0→F
3→L
F→T
45400→S
2100→I
2500→R
30→N
Range F,L,1,0,S,10000
(L-F)÷N→D
Plot T,S
1→K
Lbl 1
-.00001SI→A
.00001SI-I÷14→B
I÷14→C
AD→U
BD→V
CD→W
T+D→T
S+U→S
I+V→I
R+W→R
Plot T,S
Line
Isz K:K≤N⇒Goto 1▲

```

DOWHILE

```

DOWHILE
0→F
F→T
45400→S
2100→I
2500→R
.00001SI-I÷14→P
Range F,20,5,0,S,10000
.01→D
Plot T,S
Lbl 1
P≤0⇒Goto 2
-.00001SI→A
.00001SI-I÷14→B
I÷14→C
AD→U
BD→V
CD→W
T+D→T
S+U→S
I+V→I
R+W→R
Plot T,S
Line
B→P
Goto 1
Lbl 2
T

```

SEQUENCE

Because of the speed and screen size of the calculator, this program has been limited to plotting the first 8 approximations to $y(t)$.

SEQUENCE

Range 0,75,10,0,1500,500

1→J

Lbl 1

0→F

75→L

F→T

100→V

$2x^y(J-1) \rightarrow N$

$(L-F) \div N \rightarrow D$

Plot T,V

1→K

Lbl 2

$.1V(1-V \div 1000) \rightarrow P$

PD→C

T+D→T

V+C→V

Plot T,V

Line

Isz K:K≤N⇒Goto 2

Isz J:J≤8⇒Goto 1

LENGTH

Store the expression x^2 into Function Memory as f_1 before running this program. If you store it into another function memory, change the following program to correspond.

The program pauses after displaying the length of each segment. Press ENTER to continue.

LENGTH

0→F

1→L

2→N

$(L-F) \div N \rightarrow D$

0→T

1→K

Lbl 1

$F+(K-1)D \rightarrow A$

$A \rightarrow X$

$f_1 \rightarrow Y$

$F+KD \rightarrow B$

$B \rightarrow X$

$\sqrt{(B-A)^2 + (f_1 - Y)^2} \rightarrow S$

$T+S \rightarrow T$

K▲

S▲

Isz K:K≤N⇒Goto 1

"TOTAL":N▲

T

Note: “ x^y ” is a single character that represents *exponentiation* in Casio programs; it corresponds to “^” in the computer programs in the text.

TABLE

Store the expression $\cos x^2$ into Function Memory as f_1 before running this program. If you store it into another function memory, change the following program to correspond.

The program pauses after displaying the current value of Δy , the accumulated Δy , and the ending t . Press ENTER to continue.

```
TABLE
Rad
0→F
4→L
2xy3→N
(L-F)÷N→D
F→X
0→A
1→K
Lbl 1
f1D→Y:Y▲
A+Y→A:A▲
X+D→X:X▲
Isz K:K≤N⇒Goto 1
"ACCUMULATION":A
```

PLOT

Store the expression $\cos x^2$ into Function Memory as f_1 before running this program. If you store it into another function memory, change the following program to correspond.

```
PLOT
Rad
Range 0,4,1,0,1,1
0→F
4→L
400→N
(L-F)÷N→D
F→T
0→A
Plot T,A
1→K
Lbl 1
T→X
f1D→V
A+V→A
T+D→T
Plot T,A
Line
Isz K:K≤N⇒Goto 1
```

NEWTON

The program pauses after displaying each successive approximation. Press to continue.

NEWTON

-1→S

8→N

S→X

0→J

Lbl 1

J▲

X▲

$4Xx^y3+3X^2+2X+1\rightarrow G$

$12X^2+6X+2\rightarrow P$

$X-G\div P\rightarrow X$

Isz J:J≤N⇒Goto 1

BABYLON

The program pauses after displaying each successive value of x . Press to continue.

BABYLON

5→A

2→X

6→N

1→K

Lbl 1

$(X+A\div X)\div 2\rightarrow X:X\blacktriangleleft$

Isz K:K≤N⇒Goto 1

RIEMANN

Store the expression $\sqrt{1+x^3}$ into Function Memory as f_1 before running this program. If you store it into another function memory, change the following program to correspond.

The program pauses after displaying each successive accumulation. Press to continue.

RIEMANN

1→A

3→B

4→N

$(B-A)\div N\rightarrow D$

A→X

0→C

1→K

Lbl 1

$f_1D\rightarrow S:S\blacktriangleleft$

$C+S\rightarrow C:C\blacktriangleleft$

$X+D\rightarrow X$

Isz K:K≤N⇒Goto 1

HARMONIC

HARMONIC

100→N

0→S

1→I

Lbl 1

$S+1\div I\rightarrow S$

Isz I:I≤N⇒Goto 1

N▲

S

TAYLOR

Notice that there are *two* programs here. The first, PTAYLOR, evaluates a 17th degree Taylor polynomial at the current value of x . It is a subroutine called by the main program, TAYLOR, which refers to it as Prog Z. In the listing below, you should replace Prog Z by the name your calculator uses to refer to PTAYLOR.

```
PTAYLOR
X→S
-1→U
3→K
Lbl 2
S+UXyK÷K!→S
-1U→U
K+2→K:K≤17⇒Goto 2

TAYLOR
Range 0,3.14,1,-1.5,1.5,1
0→X
Prog Z
Plot X,S
.01→J
Lbl 1
J→X
Prog Z
Plot J,S
Line
J+.01→J:J≤3.14⇒Goto 1
```

SIRSERIES

The coefficients in the power series for $S(t)$ are stored as S[0] to S[50] and the coefficients for $I(t)$ are stored as S[51] to S[101]. So, for example, the coefficient of t^{23} in the power series for $I(t)$ is stored as S[23+51] = S[74].

```
SIRSERIES
Defm 102
.00001→A
1÷14→B
45400→S[0]
2100→S[51]
1→K
Lbl 1
0→M
0→J
Lbl 2
M+S[J]×S[K-J+50]→M
Isz J:J≤K-1⇒Goto 2
-AM÷K→S[K]
-S[K]-B×S[K+50]÷K→S[K+51]
Isz K:K≤50⇒Goto 1
```

Note: “×” represents *multiplication* in Casio programs; it corresponds to “*” in the computer programs in the text.

A.7 Sharp EL-9200/9300

SIR

The program waits each time it displays t , S , I , and R . Press ENTER to continue.

```
t=0
S=45400
I=2100
R=2500
deltat=1
Print t
Print S
Print I
Print R
Wait
k=1
Label loop
sprime=-.00001*S*I
iprime=.00001*S*I-I/14
rprime=I/14
deltas=sprime*deltat
deltai=iprime*deltat
deltar=rprime*deltat
t=t+deltat
S=S+deltas
I=I+deltai
R=R+deltar
Print t
Print S
Print I
Print R
Wait
k=k+1
If k<=3 Goto loop
End
```

SIRVALUE

```
tinitial=0
tfinal=1
t=tinitial
S=45400
I=2100
R=2500
numsteps=10
deltat=(tfinal-
    tinitial)/numsteps
k=1
Label loop
sprime=-.00001*S*I
iprime=.00001*S*I-I/14
rprime=I/14
deltas=sprime*deltat
deltai=iprime*deltat
deltar=rprime*deltat
t=t+deltat
S=S+deltas
I=I+deltai
R=R+deltar
k=k+1
If k<=numsteps Goto loop
Print t
Print S
Print I
Print R
```

Note: Be sure to distinguish between “0” (*zero*) and “0” (*Oh*).

SIRPLOT

```

tinitial=0
tfinal=3
t=tinitial
S=45400
I=2100
R=2500
numsteps=30
Range tinitial,tfinal,1,0,
    S,10000
deltat=(tfinal-
    tinitial)/numsteps
k=1
Label loop
sprime=-.00001*S*I
iprime=.00001*S*I-I/14
rprime=I/14
deltas=sprime*deltat
deltai=iprime*deltat
deltar=rprime*deltat
Line t,S,t+deltat,S+deltas
t=t+deltat
S=S+deltas
I=I+deltai
R=R+deltar
k=k+1
If k<=numsteps Goto loop
DispG

```

DOWHILE

The program waits after it displays the final t . Press to look at the graph.

```

tinitial=0
t=tinitial
S=45400
I=2100
R=2500
iprime=.00001*S*I-I/14
Range t,20,5,0,S,10000
deltat=.01
Label loop
If iprime<=0 Goto stop
sprime=-.00001*S*I
iprime=.00001*S*I-I/14
rprime=I/14
deltas=sprime*deltat
deltai=iprime*deltat
deltar=rprime*deltat
Line t,S,t+deltat,S+deltas
t=t+deltat
S=S+deltas
I=I+deltai
R=R+deltar
Goto loop
Label stop
Print t
Wait
DispG

```

SEQUENCE

Because of the speed and screen size of the calculator, this program has been limited to plotting the first 8 approximations to $y(t)$.

The program waits after plotting each approximation. Press **ENTER** to continue.

```

Range 0,75,10,0,1500,500
j=1
Label loop1
tinitial=0
tfinal=75
t=tinitial
y=100
numsteps=2^(j-1)
deltat=(tfinal-
      tinitial)/numsteps
k=1
Label loop2
yprime=.1*y*(1-y/1000)
deltay=yprime*deltat
Line t,y,t+deltat,y+deltay
t=t+deltat
y=y+deltay
k=k+1
If k<=numsteps Goto loop2
j=j+1
Wait
If j<=8 Goto loop1
DispG

```

LENGTH

The program waits after displaying the length of each segment. Press **ENTER** to continue.

```

Goto start
Label evaluate
y=x^2
Return
Label start
xinitial=0
xfinal=1
n=2
deltax=(xfinal-xinitial)/n
total=0
k=1
Label loop
xl=xinitial+(k-1)*deltax
xr=xinitial+k*deltax
x=xl
Gosub evaluate
yl=y
x=xr
Gosub evaluate
yr=y
segment=sqrt((xr-xl)^2+(yr-yl)^2)
total=total+segment
Print k
Print segment
Wait
k=k+1
If k<=n Goto loop
Print n
Print total

```

TABLE

The program waits after displaying the current value of Δy , the accumulated Δy , and the ending t . Press **ENTER** to continue.

```
Goto start
Label evaluate
y=cos t^2
Return
Label start
tinitial=0
tfinal=4
numsteps=2^3
deltat=(tfinal-
    tinitial)/numsteps
t=tinitial
accumulation=0
k=1
Label loop
Gosub evaluate
deltay=y*deltat
accumulation=accumulation+
    deltay
t=t+deltat
Print deltay
Print accumulation
Print t
Wait
k=k+1
If k<=numsteps Goto loop
Print accumulation
```

PLOT

```
Range 0,4,1,0,1,1
Goto start
Label evaluate
y=cos t^2
Return
Label start
tinitial=0
tfinal=4
numsteps=400
deltat=(tfinal-
    tinitial)/numsteps
t=tinitial
accumulation=0
k=1
Label loop
Gosub evaluate
deltay=y*deltat
Line t,accumulation,
    t+deltat,accumulation+
    deltay
accumulation=accumulation+
    deltay
t=t+deltat
k=k+1
If k<=numsteps Goto loop
DispG
```

NEWTON

The program waits after displaying each successive approximation. Press to continue.

```
start=-1
numsteps=8
x=start
n=0
Label loop
Print n
Print x
Wait
g=4x^3+3x^2+2x+1
gprime=12x^2+6x+2
x=x-g/gprime
n=n+1
If n<=numsteps Goto loop
End
```

BABYLON

The program waits after displaying each successive value of x . Press to continue.

```
a=5
x=2
n=6
k=1
Label loop
x=(x+a/x)/2
Print x
Wait
k=k+1
If k<=n Goto loop
End
```

RIEMANN

The program waits after displaying each successive accumulation. Press to continue.

```
Goto start
Label evaluate
y=√(1+x^3)
Return
Label start
a=1
b=3
numsteps=4
deltax=(b-a)/numsteps
x=a
accumulation=0
k=1
Label loop
Gosub evaluate
deltas=y*deltax
accumulation=accumulation+deltas
x=x+deltax
Print deltas
Print accumulation
Wait
k=k+1
If k<=numsteps Goto loop
End
```

HARMONIC

```
HARMONIC
n=100
sum=0
i=1
Label loop
sum=sum+1/i
i=i+1
If i<=n Goto loop
Print n
Print sum
```

TAYLOR

```

Range 0,3.14,1,-1.5,1.5,1
Goto start
Label taylor
sum=x
sign=-1
k=3
Label loopk
sum=sum+sign*x^k/k!
sign=-1*sign
k=k+2
If k<=17 Goto loopk
t=sum
Return
Label start
x=0
xl=x
Gosub taylor
yl=t
x=.01
Label loopx
xr=x
Gosub taylor
yr=t
Line xl,yl,xr,yr
xl=xr
yl=yr
x=x+.01
If x<=3.14 Goto loopx
End

```

SIRSERIES

Enter this program in matrix mode. It deletes the contents of matrices S and I , so take care to back them up if they contain important data. The first 51 coefficients in the power series for $S(t)$ are stored in $S[1,k]$ and the coefficients for $I(t)$ are stored in $I[1,k]$. Note that the elements in these matrices are numbered from 1 to 51, so the coefficient of t^{50} in the power series for $S(t)$ is $S[1,51]$.

```

dim S[1,51]
dim I[1,51]
a=.00001
b=1/14
S[1,1]=45400
I[1,1]=2100
k=1
Label loopk
sum=0
j=0
Label loopj
sum=sum+S[1,j+1]*I[1,k-j]
j=j+1
If j<=k-1 Goto loopj
S[1,k+1]=-a*sum/k
I[1,k+1]=-S[1,k+1]-b*I[1,k]/k
k=k+1
If k<=50 Goto loopk
End

```

Appendix B

Quick Reference

Formulas from Geometry and Algebra

Notation. A = area, V = volume, b = length of base, h = height (perpendicular to base), r = radius, C = circumference.

Rectangle: $A = bh$.

Triangle: $A = \frac{1}{2}bh$.

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$.

Circle: $A = \pi r^2$, $C = 2\pi r$.

Cylinder: $V = \pi r^2 h$.

Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$.

Sphere: $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$.

Pythagorean Theorem. If a right triangle has hypotenuse of length c and legs of lengths a and b , then $c^2 = a^2 + b^2$.

Quadratic Formula. The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Formulas from Trigonometry

Definition. Begin with a unit circle centered at the origin. Given the input number t , locate a point P on the circle by tracing an arc of length t along

the circle from the point $(1, 0)$. If t is positive, trace the arc counterclockwise; if t is negative, trace it clockwise. Because the circle has radius 1, the arc of length t subtends a central angle of **radian** measure t . The trigonometric functions $\cos t$ and $\sin t$ are defined as the coordinates of the point P ,

$$P = (\cos t, \sin t).$$

The other trigonometric functions are defined in terms of the sine and cosine functions:

$$\begin{array}{ll} \tan t = \sin t / \cos t & \sec t = 1 / \cos t \\ \cot t = \cos t / \sin t & \csc t = 1 / \sin t. \end{array}$$

We also have the following identities:

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ \sin(-t) &= -\sin t \\ \cos(-t) &= \cos t \\ \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos^2 t &= \frac{1}{2}(1 + \cos 2t) \\ \sin^2 t &= \frac{1}{2}(1 - \cos 2t) \end{aligned}$$

The Derivative

- A **locally-linear** function has a graph that looks approximately straight when magnified under a computer microscope.
- The **slope of the graph** at any point is the **limit** of the slopes seen under a microscope.
- The **rate of change** of a function at a point is the slope of its graph at that point, and thus is also a **limit**. Its dimensional units are (units of output)/(unit of input).
- The **derivative** of $f(x)$ at $x = a$ is the name given to both the rate of change of f at a and the slope of the graph of f at $(a, f(a))$.

- The derivative of $y = f(x)$ at $x = a$ is written $f'(a)$. The **Leibniz notation** for the derivative is dy/dx .
- $f'(a) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.
- The **microscope equation** $\Delta y \approx f'(a) \cdot \Delta x$ describes the relation between x and $y = f(x)$ as seen under a microscope.
- The microscope equation says the change in output is proportional to the change in the input. The derivative $f'(a)$ is the **multiplier**, or scaling factor.
- Functions that have more than one input variable have **partial derivatives**. A partial derivative is the rate at which the output changes with respect to one variable when we hold all the others constant.
- A function $z = F(x, y)$ of two variables also has a **microscope equation**:

$$\Delta z \approx F_x(a, b) \cdot \Delta x + F_y(a, b) \cdot \Delta y.$$

The partial derivatives are the **multipliers** in the microscope equation.

Differential Equations

- **Euler's method** is a procedure to approximate a function defined by a set of differential equations and initial conditions. The approximation is a piecewise linear function.
- The exact function defined by a set of differential equations and initial conditions can be expressed as a limit of a sequence of successive Euler approximations with smaller and smaller step sizes.
- The programs SIR, SIRVALUE, SIRPLOT, SEQUENCE, TABLE, and PLOT all compute Euler approximations.
- The function $y = Ce^{kt}$ is the solution to the initial value problem

$$\frac{dy}{dt} = ky \quad y(0) = C.$$

- In some cases, the method of **separation of variables** can be used to find a *formula* for the solution of a differential equation.

Derivatives of Basic Functions

function	derivative
$mx + b$	m
x^r	rx^{r-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$(\tan x)(\sec x)$
$\csc x$	$-(\cot x)(\csc x)$
e^x	e^x
$\ln x$	$1/x$
b^x	$\ln b \cdot b^x$

Derivatives of Combinations of Functions

function	derivative
$g(f(x))$	$g'(f(x)) \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) - g(x)$	$f'(x) - g'(x)$
$cf(x)$	$cf'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Derivatives of Inverses

The derivative of a function and the derivative of its inverse are reciprocals. Thus, when $y = f(t)$ and $t = g(y)$ are inverses:

$$\frac{dt}{dy} = \frac{1}{dy/dt}.$$

Here are the derivatives of some frequently-encountered inverse functions:

function	derivative
$\arctan(x)$	$\frac{1}{1+x^2}$
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$\frac{-1}{\sqrt{1-x^2}}$

The Integral

- A **Riemann sum** for the function $f(x)$ on the interval $[a, b]$ is a sum of the form

$$f(x_1) \cdot \Delta x_1 + f(x_2) \cdot \Delta x_2 + \cdots + f(x_n) \cdot \Delta x_n,$$

where the interval $[a, b]$ has been subdivided into n subintervals whose lengths are $\Delta x_1, \Delta x_2, \dots, \Delta x_n$, and each x_k is a sampling point in the k -th subinterval (for each k from 1 to n).

- Riemann sums can be used to approximate a variety of quantities expressed as *products* where one factor varies.
- Riemann sums give more accurate approximations as the lengths $\Delta x_1, \Delta x_2, \dots, \Delta x_n$ are made small.
- If all the Riemann sums for a function $f(x)$ on an interval $[a, b]$ get arbitrarily close to a *single number* when the lengths $\Delta x_1, \Delta x_2, \dots, \Delta x_n$ are made small enough, then that number is called the **integral** of $f(x)$ on $[a, b]$, and it is denoted

$$\int_a^b f(x) dx.$$

- The *exact* value of a quantity approximated by a Riemann sum is given by the corresponding integral. If x and $f(x)$ have units, then $\int_a^b f(x) dx$ has units equal to the product of the units of $f(x)$ times the units of x .

- **The Fundamental Theorem of Calculus.** The solution $y = A(x)$ of the initial value problem

$$y' = f(x) \quad y(a) = 0$$

is the **accumulation function**

$$A(X) = \int_a^X f(x) dx.$$

- If $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x) dx = F(b) - F(a).$$

- The integral $\int_a^b f(x) dx$ equals the **signed area** between the graph of $f(x)$ and the x -axis.
- A numerical method for estimating an integral is **efficient** if it gets accurate results with relatively few calculations. The **trapezoidal approximation** is the average of a left and a right *endpoint* Riemann sum and is more efficient than either. *Midpoint* Riemann sums are even more efficient than trapezoidal approximations.
- The most efficient method developed in this text is **Simpson's rule**. Simpson's rule approximates an integral by

$$\int_a^b f(x) dx \approx \frac{1}{6} (\text{left sum} + \text{right sum} + 4 \times \text{midpoint sum}).$$

- An **improper integral** is one that cannot be calculated directly. The problem may be that its “domain of integration” is infinite or that the integrand becomes infinite on the domain, even if the domain itself is finite. The value of the integral is obtained as a limit of ordinary integrals.

Taylor Polynomials

The **Taylor polynomial of degree n centered at $x = a$** approximating the function $f(x)$ is given by the formula

$$\begin{aligned} P_n(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k. \end{aligned}$$

In particular, The **Taylor polynomial of degree n centered at $x = 0$** approximating the function $f(x)$ is given by the formula

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k.$$

Some Taylor Polynomials

$f(x)$	$P_n(x)$
$\sin(x)$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \pm \frac{x^n}{n!} \quad (n \text{ odd})$
$\cos(x)$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \pm \frac{x^n}{n!} \quad (n \text{ even})$
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$
$\ln(1-x)$	$-\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots + \frac{x^n}{n}\right)$
$\frac{1}{1-x}$	$1 + x + x^2 + x^3 + \cdots + x^n$

Taylor's Theorem

- We say that $\varphi(x)$ has **the same order of magnitude** as $q(x)$ as $x \rightarrow a$, and we write $\varphi(x) = O(q(x))$ as $x \rightarrow a$, if there is a constant C for which

$$\lim_{x \rightarrow a} \frac{\varphi(x)}{q(x)} = C.$$

- **Taylor's Theorem.** If $f(x)$ has derivatives up to order n at $x = a$, then

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R(x),$$

where $R(x) = O((x-a)^{n+1})$ as $x \rightarrow a$. The term $R(x)$ is called the **remainder**.

- Here is a formula for the remainder:

$$R(x) = \frac{f^{(n+1)}(c_x)}{(n+1)!} (x-a)^{n+1},$$

where c_x is some number between a and x . (It is regrettable that we cannot say exactly *how* c_x is related to x —only that it exists.)

Antiderivatives

$\int f(x) dx$ denotes an **antiderivative** of $f(x)$; that is, a function whose derivative is $f(x)$.

A Note on antidifferentiation formulas. Remember that you can always add a constant to any antiderivative. Moreover, if the integrand has a break in its domain, then on different parts of the domain you can add *different* constants to the antiderivative. (See section 11.1 and the example $\int dx/x$ shown below.) We have marked with a star $\star$ any integral whose integrand has a break in its domain. (In cases where parameters are involved, we have starred the integral if there is any parameter value for which the domain of the integrand has a break.)

Why the $\star$?

Antiderivatives of Combinations of Functions

$$\int cf(x) dx = c \int f(x) dx \quad (c \text{ any constant})$$

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int G(f(x))G'(x) dx = \int G(u) du \quad (\text{for } u = f(x))$$

$$\int f(x)g'(x) dx = f(x)g(x) - \int g(x)f'(x) dx$$

Antiderivatives of Basic Functions

$$\int x^r dx = \frac{1}{r+1} x^{r+1} \quad (r \text{ any constant } \geq 0)$$

$$\star \int x^{-r} dx = \frac{1}{1-r} x^{-r+1} \quad (r > 0 \text{ and not equal to } 1)$$

$$\star \int \frac{1}{x} dx = \ln |x| \quad \text{that is: } \int \frac{1}{x} dx = \begin{cases} \ln(-x) + C_1 & \text{if } x < 0 \\ \ln(x) + C_2 & \text{if } x > 0 \end{cases}$$

$$\int e^x dx = e^x$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\star \int \sec^2 x dx = \tan x$$

$$\star \int \tan x dx = -\ln |\cos x|$$

$$\star \int \sec x dx = \ln |\tan x + \sec x|$$

$$\star \int \cot x dx = \ln |\sin x|$$

$$\star \int \csc x dx = \ln |\csc x - \cot x|$$

$$\int \frac{1}{1+x^2} dx = \arctan x$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x,$$

Other Antiderivatives

$$\begin{aligned}
\star \int \frac{dx}{ax+b} dx &= \frac{1}{a} \ln |ax+b| \\
\star \int (ax+b)^n dx &= \frac{(ax+b)^{n+1}}{a(n+1)}, \quad n \neq -1 \\
\star \int x(ax+b)^n dx &= \frac{(ax+b)^{n+1}}{a^2} \left(\frac{ax+b}{n+2} - \frac{b}{n+1} \right), \quad n \neq -1, -2 \\
\star \int \frac{x}{ax+b} dx &= \frac{1}{a^2} (ax - b \ln |ax+b|) \\
\star \int \frac{x}{(ax+b)^2} dx &= \frac{1}{a^2} \left(\ln |ax+b| + \frac{b}{ax+b} \right) \\
\star \int \frac{dx}{(ax+b)(cx+d)} &= \frac{1}{ad-bc} \ln \left| \frac{ax+b}{cx+d} \right| \\
\star \int \frac{x}{(ax+b)(cx+d)} dx &= \frac{1}{ad-bc} \left(-\frac{b}{a} \ln |ax+b| + \frac{d}{c} \ln |cx+d| \right) \\
\star \int \frac{dx}{(ax+b)^2(cx+d)} &= \frac{1}{ad-bc} \left(\frac{-1}{ax+b} - \frac{c}{ad-bc} \ln \left| \frac{ax+b}{cx+d} \right| \right) \\
\star \int \frac{x}{(ax+b)^2(cx+d)} dx &= \frac{1}{ad-bc} \left(\frac{b}{a(ax+b)} + \frac{d}{ad-bc} \ln \left| \frac{ax+b}{cx+d} \right| \right) \\
\int x\sqrt{ax+b} dx &= \frac{-2}{15a^2} (2b-3ax)(ax+b)^{3/2} \\
\star \int \frac{\sqrt{ax+b}}{x} dx &= 2\sqrt{ax+b} + \sqrt{b} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right|, \quad b > 0 \\
\star \int \frac{\sqrt{ax+b}}{x} dx &= 2\sqrt{ax+b} - 2\sqrt{-b} \arctan \sqrt{\frac{ax+b}{-b}}, \quad b < 0 \\
\star \int \frac{dx}{x\sqrt{ax+b}} &= \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right|, \quad b > 0 \\
\star \int \frac{dx}{x\sqrt{ax+b}} &= \frac{2}{\sqrt{-b}} \arctan \sqrt{\frac{ax+b}{-b}}, \quad b < 0
\end{aligned}$$

$$\begin{aligned}
\star \quad & \int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax+b}} \\
\star \quad & \int \frac{dx}{x^n \sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b} \int \frac{dx}{x^{n-1}\sqrt{ax+b}} \\
& \int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a}, \quad a \neq 0 \\
\star \quad & \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|, \quad a \neq 0 \\
\star \quad & \int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} (x\sqrt{x^2 \pm a^2} \pm a^2 \ln |x + \sqrt{x^2 \pm a^2}|) \\
& \int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left(x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right) \\
\star \quad & \int \frac{dx}{\sqrt{x^2 \pm a^2}} x dx = \ln |x + \sqrt{x^2 \pm a^2}| \\
& \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}, \quad a > 0 \\
\star \quad & \int \frac{\sqrt{a^2 \pm x^2}}{x} dx = \sqrt{a^2 \pm x^2} - a \ln \left| \frac{a + \sqrt{a^2 \pm x^2}}{x} \right| \\
\star \quad & \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \operatorname{arcsec} \frac{x}{a} \\
& \int \sin^2 x dx = \frac{1}{2} (x - \sin x \cos x) \\
\star \quad & \int \tan^2 x dx = \tan x - x \\
& \int \sin^3 x dx = -\frac{1}{3} \cos x (2 + \sin^2 x) \\
\star \quad & \int \tan^3 x dx = \frac{1}{2} \tan^2 x + \ln |\cos x| \\
\star \quad & \int \sec^3 x dx = \frac{1}{2} (\sec x \tan x + \ln |\sec x + \tan x|)
\end{aligned}$$

$$\begin{aligned}
\star \int \csc^3 x \, dx &= -\frac{1}{2} (\csc x \cot x + \ln |\csc x - \cot x|) \\
\int \sin^n x \, dx &= -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \quad n \geq 0 \\
\star \int \sin^{-n} x \, dx &= -\frac{1}{n-1} \sin^{-n+1} x \cos x + \frac{n-2}{n-1} \int \sin^{-n+2} x \, dx, \quad n > 0, n \neq 1 \\
\star \int \cos^n x \, dx &= \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx \\
\int \sin^2 x \cos^2 x \, dx &= \frac{1}{8} (-2 \sin x \cos^3 x + \sin x \cos x + x) \\
\star \int \sin^n x \cos^m x \, dx &= \\
&\quad \frac{1}{n+m} \sin^{n+1} x \cos^{m-1} x + \frac{m-1}{n+m} \int \sin^n x \cos^{m-2} x \, dx, \quad n \neq -m \\
\int x \sin x \, dx &= \sin x - x \cos x \\
\int x \cos x \, dx &= \cos x + x \sin x \\
\int x^2 \sin x \, dx &= 2x \sin x + (2 - x^2) \cos x \\
\int x^2 \cos x \, dx &= 2x \cos x + (x^2 - 2) \sin x \\
\star \int x^n \sin x \, dx &= -x^n \cos x + nx^{n-1} \sin x - n(n-1) \int x^{n-2} \sin x \, dx \\
\star \int x^n \cos x \, dx &= x^n \sin x + nx^{n-1} \cos x - n(n-1) \int x^{n-2} \cos x \, dx \\
\int x e^x \, dx &= e^x (x - 1) \\
\int x^2 e^x \, dx &= e^x (x^2 - 2x + 2) \\
\star \int x^n e^x \, dx &= x^n e^x - n \int x^{n-1} e^x \, dx
\end{aligned}$$

$$\begin{aligned}\int \sin^2 \alpha x \, dx &= \frac{x}{2} - \frac{1}{4\alpha} \sin 2\alpha x \\ \int \cos^2 \alpha x \, dx &= \frac{x}{2} + \frac{1}{4\alpha} \sin 2\alpha x \\ \int \sin \alpha x \cos \alpha x \, dx &= -\frac{1}{4\alpha} \cos 2\alpha x\end{aligned}$$

If $\alpha \neq \beta$, then:

$$\begin{aligned}\int \sin \alpha x \sin \beta x \, dx &= \frac{1}{\beta^2 - \alpha^2} (\alpha \cos \alpha x \sin \beta x - \beta \sin \alpha x \cos \beta x) \\ \int \cos \alpha x \cos \beta x \, dx &= \frac{1}{\beta^2 - \alpha^2} (\beta \cos \alpha x \sin \beta x - \alpha \sin \alpha x \cos \beta x) \\ \int \sin \alpha x \cos \beta x \, dx &= \frac{1}{\beta^2 - \alpha^2} (\beta \sin \alpha x \sin \beta x + \alpha \cos \alpha x \cos \beta x) \\ \int \cos \alpha x \sin \beta x \, dx &= \frac{1}{\beta^2 - \alpha^2} (-\alpha \sin \alpha x \sin \beta x - \beta \cos \alpha x \cos \beta x)\end{aligned}$$

Appendix C

Answers to Selected Exercises

In this appendix, we have provided partial answers for a few of the exercises, which are marked in the text with an asterisk. These answers are just intended to help you get your bearings—to let you check if you are on the right track. In most cases, your own solutions should be more detailed and provide fuller explanations.

Section 1.1

9. By 1990, the river would have become $141\frac{1}{3}$ miles shorter; therefore, it would be only $958\frac{2}{3}$ miles long.
12. $L = 991\frac{1}{12} - 1\frac{1}{3}t$ miles.
17. $S = 43493.2$, $I = 3706.8$, $R = 2800.0$.
19. b. The new threshold level is $200000/14 \approx 14286$.
 - c. Quarantine does *not* eliminate the epidemic, because S is still above the new threshold level. In other words, I will increase, at least as long as S stays above the threshold.
20. d. The susceptible population is now decreasing at the rate of 30 persons per day. Thus $S' = -30$.
 - e. Currently, $S = 15000$ persons.
21. b. When S drops below 500 the infection rate I' becomes negative, so the illness fades away.
26. There is a threshold at $S = 5000$.
27. a. R increases.
 - b. R decreases.

28. a. For I not to change, S must be 5000.
 b. For R not to change, it must be true that $I = \frac{5}{42}R$.
 e. R must satisfy the equation $\frac{47}{42}R = 45000$; thus $R \approx 40213$.

Section 1.2

5. (c), (d), and (e) are false.
 7. b. The natural domain of R consists of all real numbers except $-1/2$ and 2 .
 c. t must satisfy either $-2 < t \leq -1$ or $1 \leq t < 2$.
 9. a. The x -intercepts are at $\pm 1/\sqrt{2} \approx \pm .7071068$.
 b. $x = .7071067812\dots$
 10. b. The maximum occurs at two places: at $x = 1.57\dots$ and also at $x = 7.85\dots$
 12. b. $x = .392699\dots$
 14. c. The three graphs look like straight lines with different positive slopes. For $f(x) = 2^x$, the slope is about .69.
 20. c. $m = 9/5$; $\mu = 1/m$.
 21. d. The two values for 1970 agree, but the actual value is 6 PPM larger than the calculated value in 1980.
 22. b. The multiplier is $k = .00013$ inches per degree Fahrenheit.
 26. a. If P is the size of the colony, in grams, and P' is its growth rate in grams per hour, then $P' = .0247 P$ grams per hour.

Section 1.3

12. The output is:
- | | |
|---|----|
| 1 | 1 |
| 2 | 3 |
| 3 | 6 |
| 4 | 10 |
| 5 | 15 |

Notice that each x value is the sum of the first k whole numbers.

17. a. To the nearest tenth, $S = 133.1$, $I = 10329.7$, $R = 39537.2$.
 b. I is largest when $t = 13$.
 19. Our estimate for the largest number of new infections in one day is 4609.1; it occurs when $t = 8$. This is found by checking the rate of new infections, which is $-S'$. Be sure to print out the values of t and S' after line 10 (following the directions in exercise 18) in order to match the value of S' to the correct day.

21. I is growing most rapidly when $t = 7$ and declining most rapidly when $t = 18$. These really are the correct days; be sure you followed the directions in exercise 18 (and see the answer to exercise 19, above).

Section 1.4

3. The maximum yield is $75 \times 90^2/14$ pounds of apples, when there are 90 trees on the acre.

Section 2.1

4. $S_3(2.5) = 42295.9$
13. a. Here is part of what you should do:

Replace the line: FOR k = 1 TO numberofsteps
with the line: DO WHILE I >= 1

b. $t = 160.2$

Section 2.2

2. $y(37) = 817.98\dots$ (With 2^{18} steps, the third decimal place has stabilized at 817.987....)
11. b. The graph levels off as t increases; the right half becomes a straight line with slope 0.

Section 2.3

1. There are three roots, and the smallest is $-.7666\dots$ to four decimal place accuracy.
6. 1.478 756 512.
15. The estimate 2.236 067 977 499 79 is obtained at the fourth step. Its square is 5.000 000 000 000 001.

Section 3.1

3. The three velocity estimates are 41.83, 64.13, and 52.98.

Section 3.2

1. a. $f'(1) \approx -4.000$, using $(f(1.001) - f(.999))/.002$.
2. a. Here is a possible sequence of estimates:

Δx	1	.1	.01	.001
microscope	0	-3.96	-3.9996	-3.999996

2. a. $f'(2) \approx -.250\,000\,00$.
3. c. Q_1 is much better than Q_2 . The best Q_2 estimate is 3.012, when $h = 1/2^8 = .004$, but Q_1 gives 3.004 when $h = 1/2^4 = .063$, an h -value that is 16 times larger.
10. a. $\Delta y \approx -.25\Delta x$.
11. b. The true value is .4878048...and the microscope estimate is .4875.
The difference is about .0003.
12. d. There is a savings of 1.25 hours.

f. The total travel times are not equal. The 40 mph microscope equation predicts the trip takes 8.75 hours at 45 mph, while the 50 mph microscope equation predicts it takes 8.8 hours. (The exact formula predicts 8.89 hours!)
13. c. $f(5.3) \approx 12.12$.
18. a. 2.8
d. 2.375

2. e. Wasting 10 ft^2 means that $\Delta A = -10 \text{ ft}^2$, so $\Delta V \approx \frac{5}{2}\Delta A = -25 \text{ ft}^3$, a decrease in volume of 25 cubic feet.
4. c. $\sqrt{101} \approx 10.05$.

6. b. $\frac{d}{dx}\left(\sqrt{3}\sqrt{x} + \frac{7}{x^5}\right) = \frac{d}{dx}\left(\sqrt{3}x^{1/2} + 7x^{-5}\right) = \frac{\sqrt{3}}{2\sqrt{x}} - \frac{35}{x^6}.$
8. b. $f(x) = \frac{5x^8}{8}$ plus any constant.
9. $y' = 1 - \frac{1}{2\sqrt{x}}$, so $y'(4) = 3/4.$
11. b. $x = \sqrt{2.5}.$
13. b. $\sin .3 \approx .3$, according to the microscope equation, and $\sin .3 = .2955202$, according to a calculator. Be sure you are entering x in radians! The error in the approximation is less than .005.
15. b. $\sqrt{3592} \approx 59.933333$ according to the microscope equation, while $\sqrt{3592} = 59.933296$ according to a calculator. The difference is less than .00005.

18. d. The ball rises to a height of 201.5625 feet.

20. b. The ball never gets farther than 3 units from the center of the track.

Section 3.6

1. b. $\frac{dy}{dx} = -7 \cos(4 - 7x)$.
2. b. $4w/\sqrt{4w^2 + 1}$.
5. $f'(1) = 160$.
9. c. $f'(x) = 0$ only when $x = 0$.
10. b. $y' = -(6x - 5)(3x^2 - 5x + 7)^{-2}$, and $y' = 0$ when $x = 5/6$.

Section 3.7

1. j. $\frac{\partial(x \tan y)}{\partial x} = \tan y$, $\frac{\partial(x \tan y)}{\partial y} = x \sec^2 y$.
3. a. $V_T = \frac{R}{P}$, $V_P = -R \frac{T}{P^2}$.
6. a. $w = G(u, 2) = 2u/(3 + 2)$, so $\Delta w/\Delta u = 2/5$ always.
7. b. The uncertainty is about 1188 square feet in the total area of 103,734 square feet. The percentage error is just over 1%.
8. a. $f(4, 10) \approx 239$.

Section 4.1

1. c. 25.258 months.
4. c. Here are estimates for P in 1980 (using different step sizes Δt):

Δt	1	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
P	4.00	4.05	4.07	4.08	4.09	4.09	4.09

12. b. Here are estimates that show $Q(20) \approx 29.47^\circ\text{C}$:

Δt	1	2^{-2}	2^{-4}	2^{-6}	2^{-8}	2^{-10}
Q	27.66	29.00	29.35	29.44	29.47	29.47

15. There are 112 susceptibles after 40 days. The largest number of people are infected when $t = 6.33$ days. At that time, there are 8000 susceptibles.

5. a. $C = 286$.
9. a. The doubling time is $t = .69/k$.
14. b. $k \approx .3364$; you need to explain *why* this is the value.

3. d. $3.0e^{2t}$.
4. a. $\frac{\partial(e^{xy})}{\partial x} = ye^{xy}, \quad \frac{\partial(e^{xy})}{\partial y} = xe^{xy}.$
8. c. The formula predicts that the population of Poland will be about 44.9 million people in 2005.

1. b. 3 f. 8 o. $1/2$
4. a. $1/x$.
6. The intensity is cut in half when $s \approx 1.98$ feet.
22. a. The inverse is $f(x) = x^2$, and $f'(x) = 2x$. Since $g(100) = 10$, $g'(100) = 1/f'(10) = 1/20$.

1. b. $F(t) = \frac{1}{7}t^7 - \frac{8}{6}t^6 + 22\pi^3t$.
2. $G(5) = 3.3$.
8. a. The accumulated change Δy is 0 as t increases from 0 to 1; however, $\Delta y = 4$ as t increases from 1 to 2.

1. b. $60x^{11}(\pi - \pi^2x^4) - 4\pi^2x^3(5x^{12} + 2)$.
f.
$$\frac{-4\pi^2x^3(5x^{12} + 2) - 60x^{11}(\pi - \pi^2x^4)}{(5x^{12} + 2)^2}$$
.
8. The current daily per capita energy consumption is 8×10^5 BTU/person.
This is *falling* at the rate of 10^3 BTU/person per year.
22. a. $f''(x) = 9e^{3x-2}$.

Section 6.3

1. a. 39.
4. c. $\int_0^3 \frac{\cos x}{1+x^2} dx = .6244\dots$
9. b. error $< .00038$. d. $\int_1^2 e^{-x^2} dx = .135\dots$
13. b. The average value of $\sin x$ on $[0, 2\pi]$ is 0.

Section 6.4

4. a. $A'(X) = \cos(X)$.
5. There are five critical points in the interval $[0, 4]$. The first is a local maximum at $\sqrt{\pi/2}$.
10. a. $474\frac{2}{3}$.
11. b. $\frac{1 - \cos(\alpha\pi)}{\alpha}$.
12. a. $y = \frac{1}{3}x^3 - x^4$.

Section 7.2

1. b. $\omega = 1/5$.
7. a. $2/\pi$.
9. $\varphi = -\pi/2$.
12. b. $\pi/3$.
20. $F(\varphi) = \cos(\varphi)/2$.
24. The phase *difference* is $\varphi = 2\pi$. A phase *shift* of $\Delta x = 2\pi/3$ makes the two graphs coincide.

Section 7.3

2. b. The periods are the same, but the impulse solution has larger amplitude and is phase-shifted with the respect to the text solution.
7. b. $\frac{1}{2}p^2 + \frac{1}{2}b^2a^2$.
10. a. Solve the differential equations numerically. Then you should find that, in 10 cycles, $\Delta t \approx 13.7$ sec; thus the frequency is about $10/13.7 \approx .73$ cycles per second.

24. The threshold impulse value is $p = \sqrt{2}$.
28. The period is 108.7 months (because the time unit in the original problem was 1 month).

Section 8.1

3. b. There are three equilibrium points, at $(0, 0)$, $(200, 0)$, and $(0, 200)$.
4. a. The three equilibrium points from question 3 reappear here, plus a fourth, at $(1000/7, 1000/35) \approx (142, 28)$.
- c. The species X grows to 200 while the species Y disappears. Nothing like this happens in question 3!
- d. X and Y coexist at the equilibrium $(1000/7, 1000/35)$, but this equilibrium is unstable. If x and y are changed even slightly from $1000/7$ and $1000/35$, they will gradually drift farther and farther away from these values. Eventually, either x or y becomes 0—one species disappears entirely.
- f. The equilibrium points at $(200, 0)$ and $(0, 200)$ are both stable; the other two are unstable.
8. c. The trajectory is a spiral, but it moves in very slowly.
- d. It will appear that, after an initial burst of infection, the disease has been effectively eliminated.

Section 8.2

2. b. To get the local linearization at $R = d/c$, $F = a/b$, introduce the variables $r = R - d/c$, $f = F - a/b$. Then the differential equations are

$$r' = -\frac{bd}{c}f \quad f' = \frac{ac}{b}r.$$

Notice that the *nonlinear* parts of the differential equations have been removed. These are $-brf$ (in the equation for r') and crf (in the equation for f').

Section 8.3

4. a. $m = \frac{(d - a) \pm \sqrt{(a - d)^2 + 4bc}}{2b}$.
12. $a = -d$.

Section 8.5

2. b. The equilibrium points are $(x, y, z) = (0, 0, 0)$, $(1500, 250, 0)$, $(1000, 200, 4)$, $(1000, 0, 0)$, and $(0, 100, 0)$.
 c. For the case $(x, y, z) = (1000, 200, 4)$, let

$$x = u + 1000$$

$$y = v + 200$$

$$z = w + 4$$

and substitute these values into the differential equations to get the *localized* system

$$u' = -u - .001 u^2 + 2v + .002 uv - 100w - .1uw$$

$$v' = .2u - 2v + .001uv - .01v^2$$

$$w' = .004u + .001uw.$$

The *linearized* system derived from this is

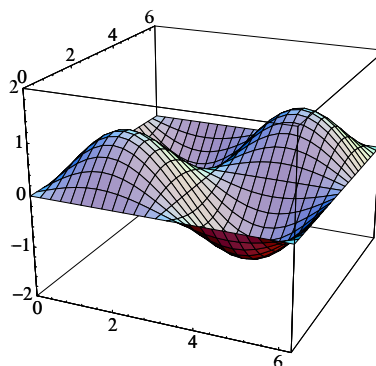
$$u' = -u + 2v - 100w$$

$$v' = .2u - 2v$$

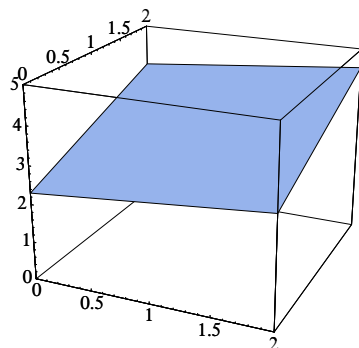
$$w' = .004u.$$

Section 9.1

1. a. Graph of $z = \sin x \sin y$
 There are two minima,
 two maxima,
 and one saddle on the domain.



9. a.



- b. The graph is a plane. Vertical slices in the y direction are straight lines with slope .8, while those in the x direction are straight lines with slope .3.

-

7. a. $\Delta z = 122$.

13. $z = -4(x - 2) + 5(y + 9) = -4x + 5y + 53$.
15. a. $L_1(1, 0) = 15$.
b. $\Delta L_1 / \Delta x = 3$.
24. c. The difference is 10, independent of the formula you use.
26. a. y must increase by 125.
34. $\Delta z = 12\Delta x - 8\Delta y$.
40. a. $\frac{\partial Q}{\partial x}(27.6, 31.9) \approx -\frac{5}{6}$.
b. $Q(27.6, 31.9) \approx 15.45$ and $Q(27, 31.9) \approx 15.95$.
44. b. At $(35.9, 25.564)$, H is about 2.796 units larger than it is at $(35, 26)$.
46. a. The error in the calculated volume will be $500\Delta x + 625\Delta y$ cubic inches.
47. A 1% error in the measurement of the base causes a 2% error in the volume, but a 1% error in the measurement of the height causes only a 1% error in the volume.
57. a. $z = 0$.

4. The maximum is found at $(x, y) = (25/8, 15/8)$, as long as $a \geq 25/8$. When $2 \leq a \leq 25/8$, the maximum is at $(x, y) = (a, 5 - a)$, and when $0 \leq a \leq 2$, the maximum is at $(x, y) = (a, 3a/2)$.
11. The maximum is $800/27$; it occurs where $(x, y) = (20/3, 2/3)$.
18. X should be at A . Does this surprise you?

20. Remarkable as it may seem, there is an entire line segment of solutions to this problem in the m, b plane. One end of the line is near $(m, b) = (.67, 2.3)$, the other is near $(m, b) = (1, 1)$.

Section 10.2

1. a. $\int \frac{\sin(x)}{x} dx \approx x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!}.$
6. c. $\sin(3x) \approx -\frac{3(x-\pi)}{1!} + \frac{3^3(x-\pi)^3}{3!} - \frac{3^5(x-\pi)^5}{5!} + \frac{3^7(x-\pi)^7}{7!}.$
 This suggests that $\sin(3x) = -\sin(3(x-\pi))$ (do you see why?), and indeed, this is true.
12. $1 - \frac{1}{3}x^2 + \frac{2}{9}x^4 - \frac{14}{81}x^6.$
14. $\text{erf}(.3) = .3286267 \dots$
15. a. $\tan(x) \approx x + x^3/3 + 2x^5/15.$
23. a. 1 b. $1/2$ d. $1/6$
29. c. As $x \rightarrow \infty$, $\frac{\sqrt{1+x^2}}{x} = \sqrt{\frac{1+x^2}{x^2}} = \sqrt{\frac{1}{x^2} + 1} \rightarrow 1$; hence $\sqrt{1+x^2} = O(x)$ as $x \rightarrow \infty$.

Section 10.3

2. a. The Taylor series converges for $-1 \leq x < 1$.
3. a. $u - u^2/2 + u^3/3 - u^4/4 + u^5/5 + \dots$
5. a. You have to go out at least past the term of degree 4909; exactly how far may depend on the computer or calculator you use.
10. $3 + 4 + 4 + 4 + 4 + 1 = 20.$
11. The old way takes roughly 13 times as long.

Section 10.4

1. a. $y = a_0 \left(1 + \frac{x^2}{1} + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots + \frac{x^{2n}}{n!} + \dots \right).$
5. a. $J_1(x) = \frac{x}{2} - \frac{1}{2!1!} \left(\frac{x}{2}\right)^3 + \frac{1}{3!2!} \left(\frac{x}{2}\right)^5 + \dots + \frac{(-1)^n}{(n+1)!n!} \left(\frac{x}{2}\right)^{2n+1} + \dots$

Section 10.5

4. a. Converges. c. Diverges.
5. a. We must have $\frac{n+1}{(n+1)^2+1} < .5 \times 10^{-6}$. This will happen, for example, if $n = 2 \times 10^6$. Finally,

$$\sum_{n=1}^{2 \times 10^6} (-1)^n \frac{n}{n^2+1} = -.269610 \dots$$

and this is the sum of the *infinite* series as well, to six decimal places accuracy.

8. c. $9.21044 < S_{10000} < 10.21035$.
- d. It would take approximately 10^{87} cards—a number which is on the same order of magnitude as the number of atoms in the universe!
10. c. $R = 1$.
11. a. $R = 1$. b. $R = 2$. c. $R = 1$. d. $R = \infty$. e. $R = 0$.

Section 10.6

1. a. $I_0 = 2$, and $I_1 = \pi$.
- c. $I_2 = \pi^2 - 4$, $I_3 = \pi^3 - 6\pi$, and $I_4 = \pi^4 - 12\pi^2 + 48$.
- d. $.00131 + .98260x + .05447x^2 - .23379x^3 + .03721x^4$, and the maximum difference occurs at the endpoints.
3. a. $I_0 = \arctan(2) = 1.10715$, and $I_1 = \frac{1}{2} \ln 5 = .804719$.
- c. $I_2 = 2 - \arctan(2)$, $I_3 = 2 - \frac{1}{2} \ln 5$, $I_4 = \frac{2}{3} + \arctan(2)$.

Section 11.1

2. The inverse of $y = 1/x^3$ is $y = 1/\sqrt[3]{x}$.
7. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$.
10. e. $\int \left(7y + \frac{1}{y}\right) dy = \frac{7}{2}y^2 + \ln y$.
- g. $\int (5 \sin w - 2 \cos w) dw = -5 \cos w - 2 \sin w$.

11. a. $F(x) = 7x + 12$.

16. b. $C = \ln 2$.

21. a. $F'(x) = x^2 + x^3$.

d. $F'(x) = 2x \cos(x^2)$.

22. b. $\int_0^5 \frac{t}{\sqrt{1+t^2}} dt = \sqrt{26} - 1$.

23. e. The *exact* value is $\sqrt{\pi}/2$.

24. $71\frac{1}{4}$.

27. c. The two areas are equal.

Section 11.2

1. c. $2e^{\sqrt{x}}$.

g. $-\ln |\cos x|$.

2. b. $\ln(e+1) - \ln(2)$.

d. $-\frac{1}{2} \ln 5$.

4. b. $F(x) = \frac{1}{42} (1+x^3)^{14} + 3\frac{41}{42}$.

Section 11.3

1. a. $\sin x - x \cos x$.

2. $\int (\ln x)^2 dx = x [(\ln x)^2 - 2(\ln x) + 2]$.

8. b. $1 - 5e^{-4}$.

21. b. $n\pi/2$.

Section 11.4

1. $y = Ce^t - 5$.

2. c. $y = \pm \sqrt{x^2 + C}$.

3. b. $Q(20) = 20 + 70e^{-1 \cdot 20} = 29.47^\circ\text{C}$.

8. $Q = [A^{1-p} - (p-1)kt]^{\frac{1}{1-p}}$. For supergrowth to occur, we must have $p > 1$, and then $\tau = A^{1-p}/k(p-1)$.

9. a. $\alpha = \frac{1}{3}, \beta = -\frac{1}{3}$.

10. a. $\int \frac{3dx}{(x-1)(x+2)} = \ln \left| \frac{x-1}{x+2} \right|$

Section 11.5

1. a. $\frac{1}{2} \arcsin(2x)$.

h. $-\frac{1}{2} \ln \left| \frac{2 + \sqrt{4 + x^2}}{x} \right|$.

4. a. $\ln |x - 1 + \sqrt{x^2 - 2x - 8}|$.

5. b. $\tan u = \frac{\sqrt{x^2 - a^2}}{a}$.

Section 11.6

3. a. When
- $n = 2^2$
- , the sum is
- $.\underline{7468}26\dots$
- and the underlined digits have all stabilized.

Section 11.7

1. a. $\int_{-\infty}^0 e^x dx = 1$.

c. $\int_0^1 \frac{dy}{y^2} = \infty$.

Section 12.1

5. The probability of exactly 50 heads in 100 tosses of a coin is

$$\begin{aligned} \frac{100!}{2^{100}(50!)^2} &\approx \frac{\sqrt{2\pi} \cdot 100^{100} \cdot \sqrt{100} \cdot e^{-100}}{2^{100} \cdot 2\pi \cdot 50^{100} \cdot 50 \cdot e^{-100}} \\ &= \frac{\sqrt{2\pi} \cdot 100^{100} \cdot 10 \cdot e^{-100}}{2\pi \cdot (2 \cdot 50)^{100} \cdot 50 \cdot e^{-100}} = \frac{1}{5\sqrt{2\pi}} \approx .0797885. \end{aligned}$$

Incidentally, *Mathematica* gives the exact value:

$$\frac{12\,611\,418\,068\,195\,524\,166\,851\,562\,157}{158\,456\,325\,028\,528\,675\,187\,087\,900\,672} \approx 0.0795892.$$

The ratio

$$\frac{\text{Stirling's value}}{\text{exact value}} \approx 1.0025$$

tells us Stirling's approximation is within $\frac{1}{4}\%$ of the exact value.

Section 12.2

1. The following table gives the probability of each outcome.

outcome	<i>H1</i>	<i>H2</i>	<i>H3</i>	<i>H4</i>	<i>H5</i>	<i>H6</i>	<i>TT</i>	<i>TH</i>
probability	1/12	1/12	1/12	1/12	1/12	1/12	1/4	1/4

6. Theoretical frequencies: 228, 212, 98, 30, 7, 1.

Section 12.3

13. The power spectrum has a single peak of height 1 at $\omega \approx 1$
15. A new peak, of height 1, appears at $\omega \approx 7/3$.
20. The Riemann sum is $-.3(2\sin^2(2\pi/5) + 2\sin^2(\pi/5)) = -.75$.
21. -1 . Since $h(x)$ is periodic with period $x = 1.5$, the interval $[0, 30]$ contains 20 periods of h . The integral of h over $[0, 30]$ is therefore 20 times its integral over $[0, 1.5]$.

Section 12.4

3. a. $2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin nx}{n}$

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