# **User Guide**

## **©** Goal of the Game

Your challenge is to choose numbers between 0 and 1 so that your number lies **between two of the computer's orange points** on the number line — with no other points in the interval bounded by the two target orange points.

When you succeed, those two computer points disappear and you score one point!

## **How to Play**

### 1. Choose the Type of Numbers

Use the **Type** menu to pick whether numbers will appear as:

- o Fractions (like 2/5 or 3/4),
- o **Decimals** (like 0.40 or 0.75), or
- Both, which mixes the two formats.

### 2. Watch the Computer's Picks

- o Each round, the computer places **two orange points** on the top number line.
- o Their values also appear in the **Turn Log** on the right.

### 3. Make Your Guess

Type your number into the input box.

You can enter either:

- a fraction (e.g., 3/5), with a denominator ≤ 999, or
- a **decimal** (e.g., 0.625), with up to three decimal places.
- Click Submit Turn or press Enter.

### 4. Scoring

- If your blue point lands between two computer points and those are the nearest on both sides, they blink for 2 seconds, disappear, and you earn +1 point.
- If not, nothing is removed and play continues.

#### 5. Rounds

- There are 10 turns total.
- Your current turn number and score appear above the number line.

#### 6. Start Over

Click New Game to reset everything and begin again.

## **Comparison Lines**

Two extra number lines below the main one help you visualize fractions:

Each one has tick marks determined by its Comparison menu (values 3–25).

• Their labels are **always fractions** (e.g., 1/8, 1/4, 3/8, 1/2, etc.), even if the game mode is in decimals.

Use these to compare how your number fits on a finer or coarser scale.

## 🎀 Visual Notes

- Orange circles = Computer's picks
- Blue circles = Your picks
- Labels alternate above and below the main line to stay readable.
- Points that appear or disappear blink for 2 seconds to highlight changes.

## **Teacher Guide**

This interactive game reinforces:

- Understanding of the number line between 0 and 1,
- Equivalence and ordering of fractions and decimals, and
- Conceptual precision in locating values.

It helps students see that fractions and decimals represent the same continuum and builds intuition about **relative size** and **density of rational numbers**. Successful play requires students to think about fraction operations (averaging fractions, common denominator, etc.) and place value.

# Game Design Highlights

- The computer's first four picks are simple fractions (denominators ≤ 12) to keep early rounds accessible.
- Later rounds expand denominators up to 25 and include decimals if chosen.
- Students' input can be **fractions (a/b)** or **decimals**, with live validation (no duplicates, values < 1, etc.).
- The **Turn Log** records all moves for discussion and review.
- Visual design emphasizes number sense:
  - Clear 0–1 boundary
  - Tenths on the main line
  - Fractional comparisons below

# Classroom Uses

### 1. Guided Exploration

- Demonstrate how numbers like 1/2, 3/4, and 0.6 fit along the same line.
- Discuss which values are "between" others.

### 2. Individual or Small-Group Play

- Students play independently or in pairs, explaining reasoning aloud.
- Encourage discussion of strategies: "Why did you choose that number?"

## 3. Concept Reinforcement

- Use rounds to explore **equivalent fractions** (e.g., 2/4 vs. 1/2).
- Compare decimal and fraction representations.

### 4. Assessment / Reflection

- Review Turn Logs to assess understanding of number placement.
- Ask students to explain one of their successful moves.

# Teaching Tips

- Before play with the final mixed format version, review how to convert between fractions and decimals.
- Use Comparison lines (e.g., 1/8 and 1/16) to visualize finer differences.

#### **Fraction Discussions**

- Ask students for their strategies:
  - Some students will note that the average of two fractions must be in the middle.
  - o Some will try adding 1 to the smaller fraction's numerator to make it a bit bigger.
  - Some may try subtracting 1 from the larger fraction's numerator to make it a bit smaller.
  - Some may try subtracting 1 from the smaller fraction's denominator to make it a bit bigger.
  - Some may try adding 1 to the larger fraction's denominator to make it a bit smaller.
- If students are not coming up with strategies, here are some possible prompts:
  - o How are you making fractions bigger? How are you making them smaller?
  - What can you do to the numerator to make a fraction bigger? Smaller?
  - o What can you do to the denominator to make a fraction bigger? Smaller?
  - How do you know your guess is between the two fractions? How can you compare them?
- The above strategies may not work:
  - o The average may have already been plotted and can't be reused.
  - The adjustments to a smaller fraction may make it not just bigger, but bigger than the larger fraction.
  - A student may make a finer adjustment by changing a numerator by less than 1. For example, to find a value between 5/7 and 3/4, a student might get a common denominator and end up with 20/28 and 21/28. So, to get a value between these, they choose 20.5/28. This is a correct answer, but the program requires whole number numerators and denominators. They can be asked how to turn this into an equivalent fraction with only whole numbers.

• Note that going into the middle of an interval is not always the best strategy for setting up success in future rounds of a game, so different ways to get near other points will be interesting to explore.

#### **Decimal Discussions**

The decimal version is simpler than the fraction version. The important issue to emphasize in comparing numbers left to right focusing on the bigger digit in the bigger place values. Help students see why 0.2 is not smaller than 0.197 (even though 197 is bigger than 2, 197/1000 is not bigger than 2/10 or 200/1000).

For the mixed decimal and fraction version, students will need to convert in both directions.

## **B** Possible Extensions

- Discuss why the line can never be "full" an introduction to the density of rational numbers. We can always find a new fraction between any two distinct ones (once we remove the restriction about the denominator being less than 999).
- Discuss how the decimal version does not include some simple fractions such as 1/3, which require infinitely repeating decimal.
- A student might invent their own fraction addition trick and for two fractions a/b < c/d propose that "adding across"  $-\frac{a+c}{b+d}$  produces a value between the two. This may not be a correct way to add or average fractions, but it is a cool exploration to see if it serves the purpose does it always produce a value between the two original fractions? Here is the start of a proof:
  - We are wondering if  $\frac{a+c}{b+d}$  has to be bigger than a/b. Let's work backwards first:
    - $\frac{a}{b} < \frac{a+c}{b+d}$ . Multiply both sides by both denominators:
    - a(b+d) < b(a+c). Distribute and simplify.
    - ab + ad < ab + bc
    - ad < bc. Divide by bd.
    - $\frac{a}{b} < \frac{c}{d}$ . This is our original statement, so if we reverse the steps, we see that if  $\frac{a}{b} < \frac{c}{d}$  then it must be true that  $\frac{a}{b} < \frac{a+c}{b+d}$ .
    - Follow the same process comparing our interesting  $\frac{a+c}{b+d}$  fraction with  $\frac{c}{d}$  and the proof is complete.

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