Optimizing Area, Perimeter, and Surface Area **Name _____**

One of the most common applications of mathematics is to optimize a situation, which means to find maximum or minimum possible solution. For example, if you are designing a car, you might want it to be most fuel efficient, which means to use the minimum amount of energy possible to go a specific distance. Or you might want it to maximize how fast it can travel or to minimize the cost to produce it.

For each of the following problems, be thorough. Consider numerous examples for each problem. Count all surfaces in all directions.

1. Use 4 cubic centimeters (unit cm^3). For each question, all of the cubes have to be used, have to be part of one continuous object, and each cube has to share at least one full square face with at least one other cube.
    a. Put the cubes together in one shape and determine the surface area (including those resting on the table).

    b. What shape will have the **greatest possible surface** area for your set of 4 cubes? Describe it and report both the volume and surface area.

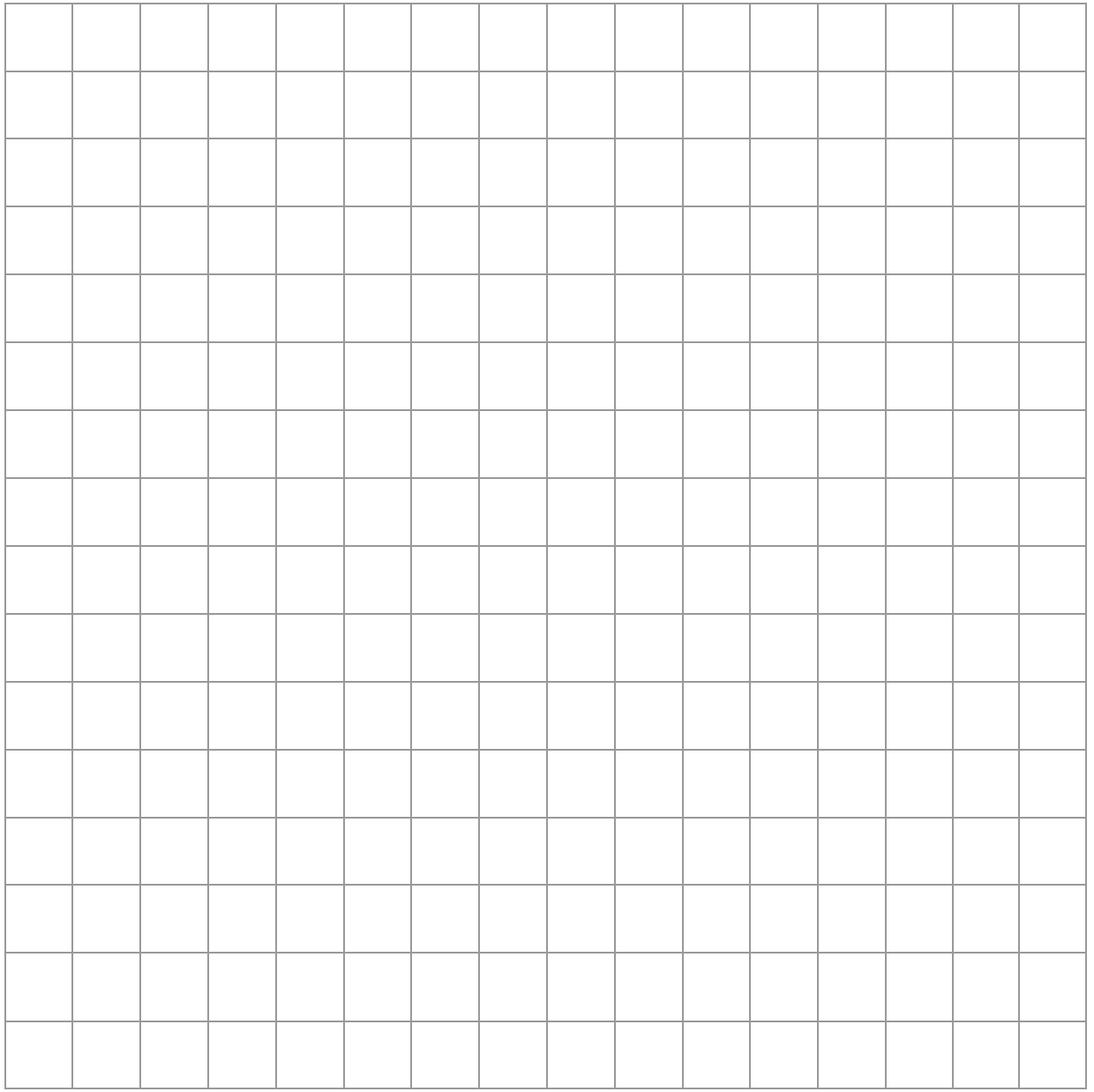
    c. What shape will have the **least possible surface** area for your set of 4 cubes? Describe it and report both the volume and surface area.

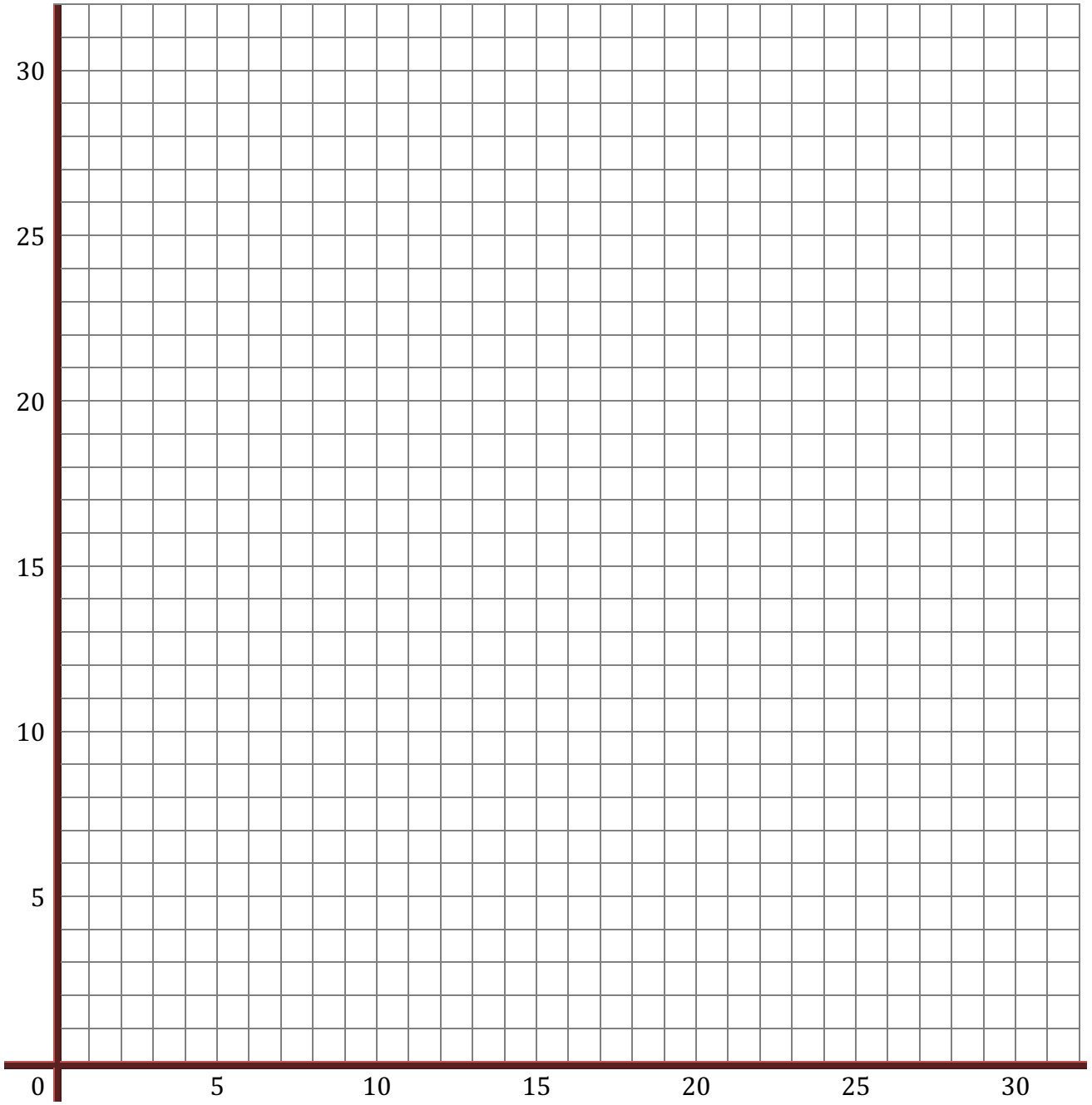
2. You have been given a handful of cubic centimeters (unit cm^3). For each question, all of the cubes have to be used, have to be part of one continuous object, and have to share at least one full square face with at least one other cube.
    a. What shape will have the **greatest possible surface** area for your handful of cubes? Describe it and report both the volume and surface area.

    b. What shape will have the **least possible surface** area for your handful of cubes? Describe it and report both the volume and surface area.

3. You have a **rectangle** with a **perimeter of 32 cm**.
    a. Fill the table below with many possible dimensions for your rectangle. **Don't limit yourself to whole numbers.** You can use the centimeter grid at right to draw some of your rectangles.
    b. Graph the values on the graph on the next page. **Label** the x-axis "width" and the y-axis "height".

Width	Height	Perimeter	Area
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	
		32	




30
25
20
15
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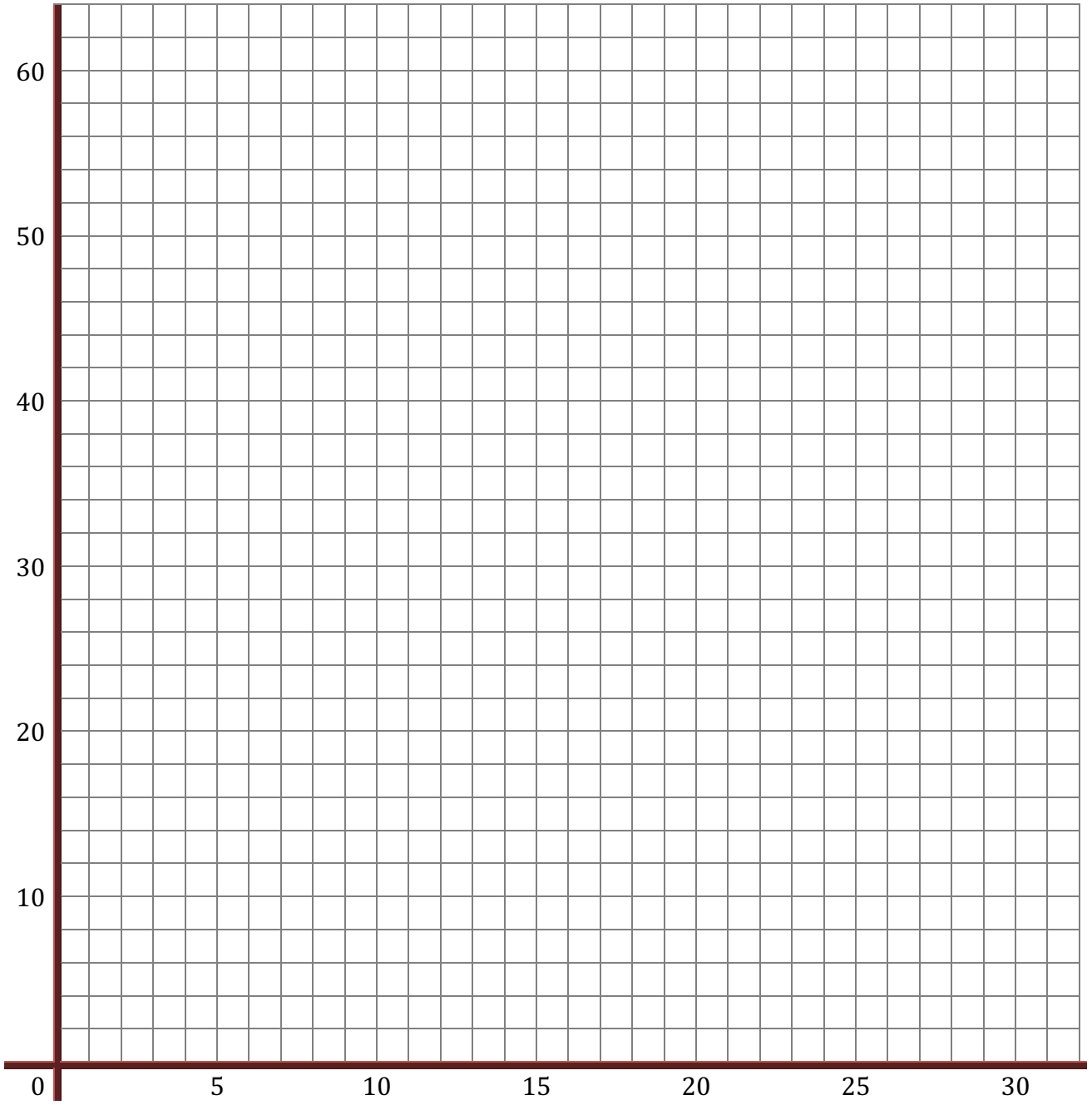
20
25
30

4. Return to the first page and complete the area column for each rectangle.
5. Plot points on the graph below with your *x*-axis being the **width** of each rectangle and the *y*-axis being that rectangle's **area**. Label both axes. Note that the *y*-axis has a different scale with each box 2 units high (but still only one unit wide).



6. Studying the graph for problem 5., determine the dimensions of the rectangle that had the greatest area. If you are not sure that rectangle's point is graphed, use the points you have to explore the region where the maximum seems to be and find it.

7. Why is that rectangle the one with the most area for the fixed amount of perimeter?

8. With any graph, we should ask, "is it appropriate to connect the points?" If the cases can only be whole numbers, we shouldn't, but lengths can be any value (whole or with a fractional or decimal part) and so we should connect the plotted points in a way that follows the pattern of those already graphed. That connecting line or curve means there are an infinite number of other possibilities! Connect the points in your two graphs above. If you think you are not sure how to draw the graph across the gaps, plot more points that fit the requirements of the data.

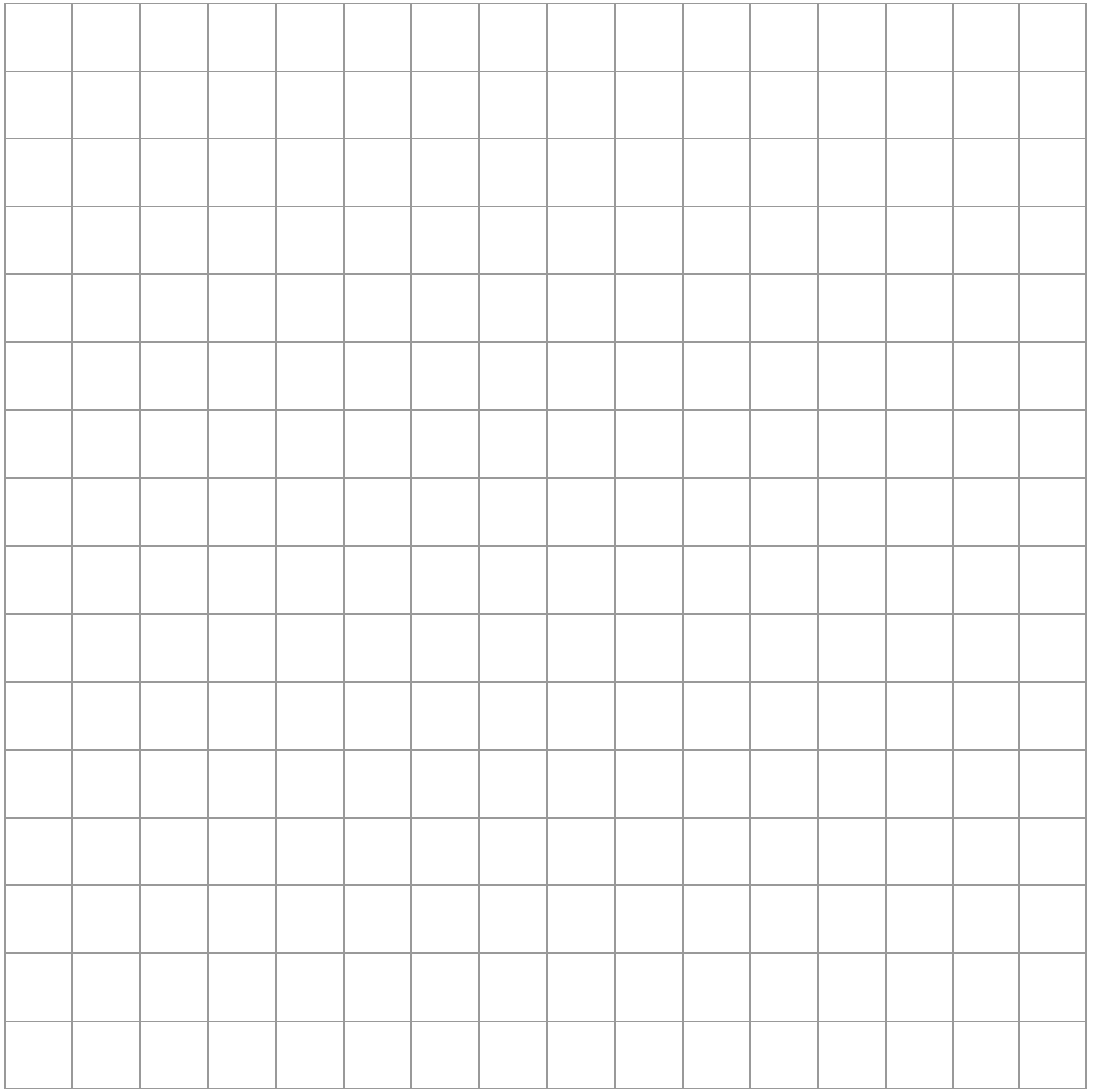
9. With any graph, we need to ask, "is it appropriate to extend the graph beyond the points?" Conencting points, as above, is called interpolation (filling in *within*). Extending a pattern is called extrapolation (extending *beyond*). In the case of extrapolation, we need to be careful not go too far (e.g., we would not want to claim that a rectangle can have sides with negative lengths). Extend the above graphs until you think they can no longer represent a rectangle. Do your graphs end? If so, where? If not, why not?

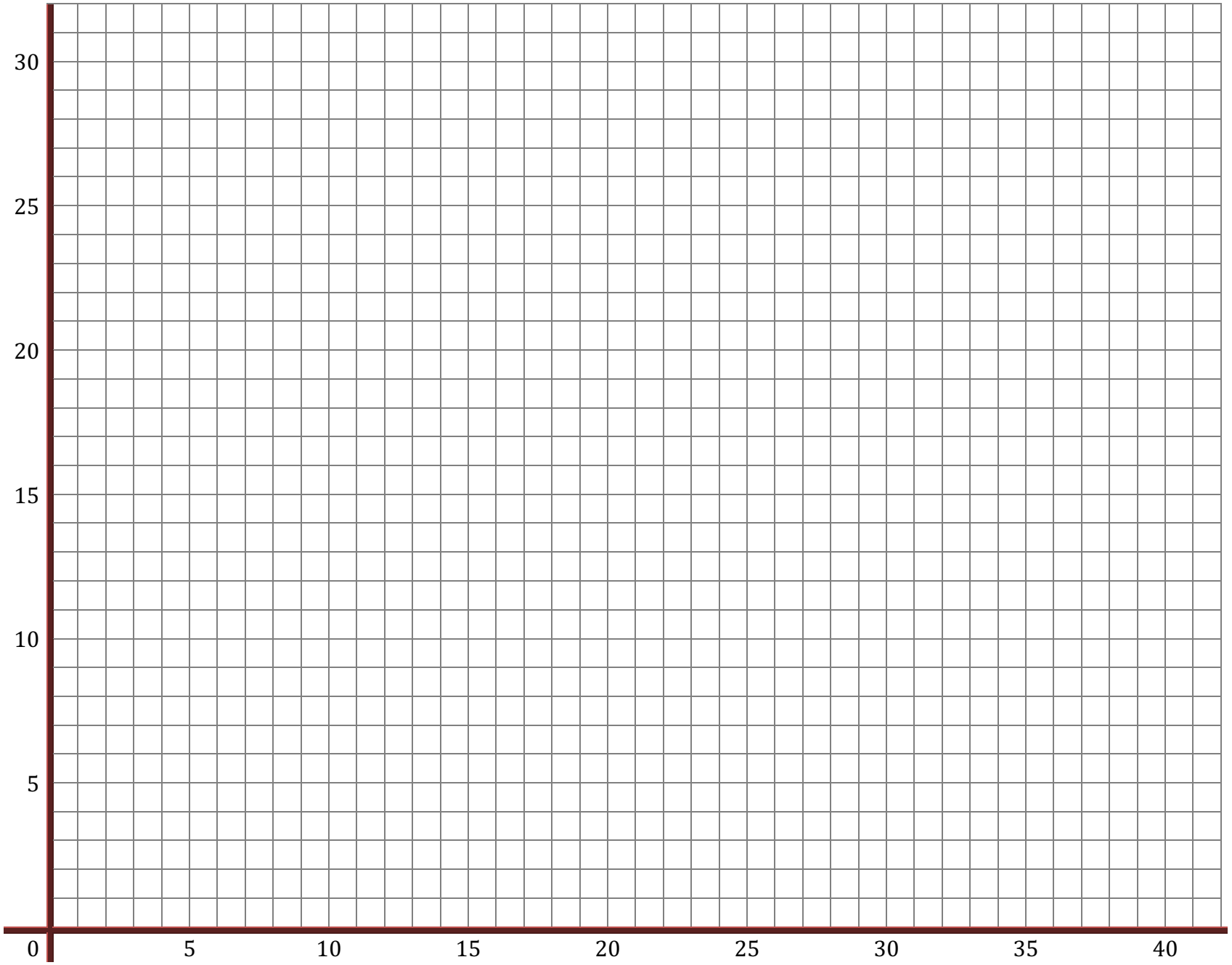
10. Which rectangle has the <u>least</u> possible area for a perimeter of 32 cm? How can you convince someone that your answer is the actual minimum possible?

11. If you had 32 meters of fence, what rectangle would enclose the greatest amount of land? Can you think of another shape that might hold even more land for the same amount of fencing? Show some calculations to find the area for your proposed shape.

12. You have a rectangle with an **<u>area</u>** of **20 cm²**.
    a. Fill the table below with many possible dimensions for your rectangle. **Don't limit yourself to whole numbers.** You can use the centimeter grid below to draw some of your rectangles.
    b. Graph the values on the graph on the next page. **Label** the x-axis "width" and the y-axis "height". Note the new scale and try to use as much of the graph as possible (practice your interpolation and extrapolation skills!).

Width	Height	Perimeter	Area
			20
			20
			20
			20
			20
			20
			20
			20
			20
			20
			20
			20
			20
			20
			20




30
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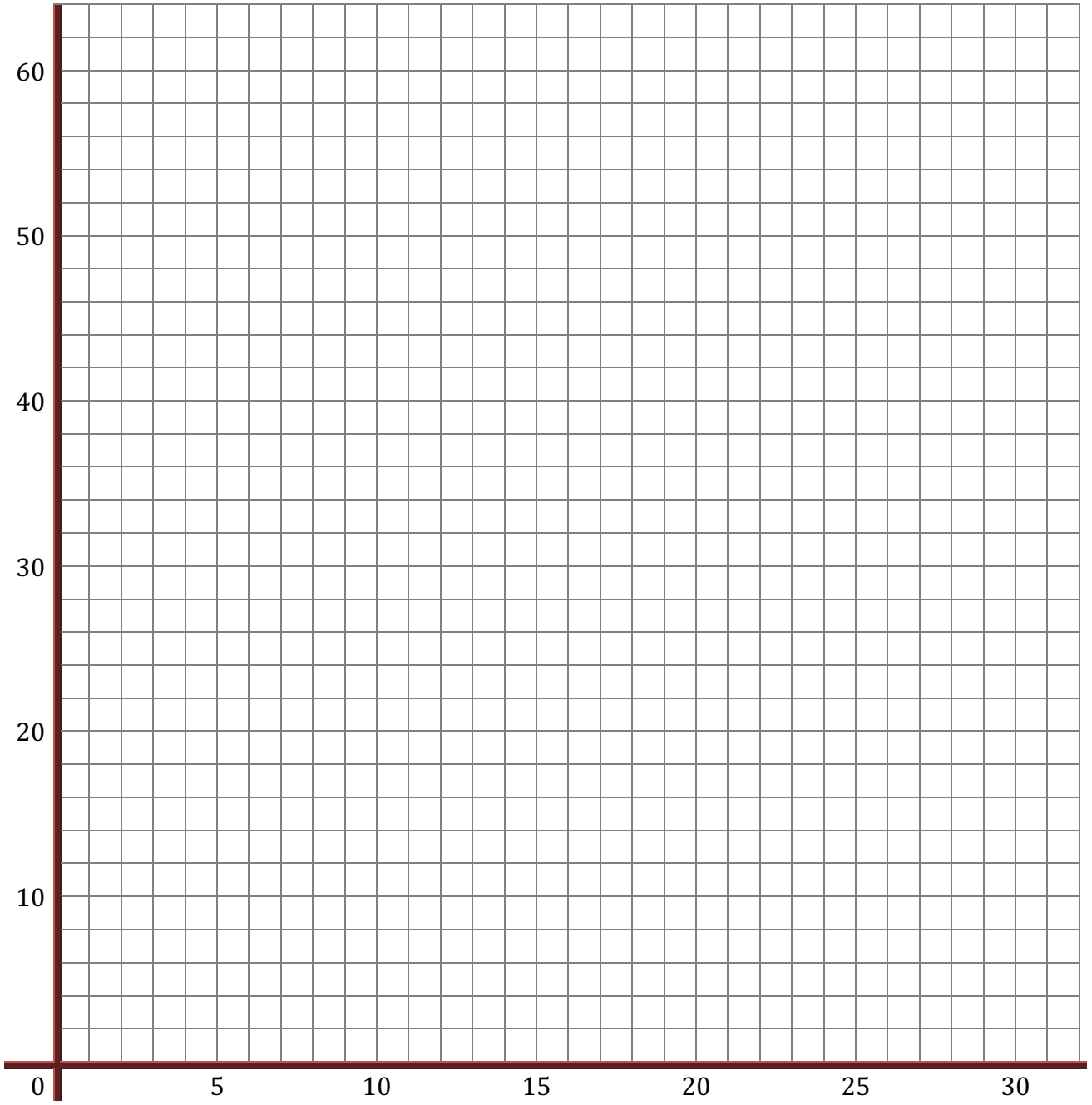
35
40

13. Return to the area chart two pages ago and complete the perimeter column for each rectangle.
14. Plot points on the graph below with your *x*-axis being the **width** of each rectangle and the *y*-axis being that rectangle's **perimeter**. Label both axes. Note that the *y*-axis has a different scale with each box 2 units high (but still only one unit wide).



15. Studying the graph for problem 14., determine the dimensions of the rectangles that have the **greatest** perimeter and the **smallest** perimeter for that fixed area.
    a. Is there a maximum possible perimeter for a rectangle with an area of 20 cm^2? If so, what is it? If not, why not?

    b. Is there a minimum possible perimeter for a rectangle with an area of 20 cm^2? If so, what is it? If not, why not?

16. You have a rectangular box with a volume of 64 cm^3. Find some possible dimensions. Do not limit yourself to whole numbers.

Width	Depth	Height	Volume	Surface Area
			64	
			64	
			64	
			64	
			64	
			64	

a. What is the smallest possible surface area for a box with a volume of 64 cm^3? Explain.

b. What is the largest possible surface area for a box with a volume of 64 cm^3? Explain.

c. What is the smallest possible surface area for a box with a volume of 1000 cm^3? Explain.

d. What is the smallest possible surface area for a box with a volume of 100 cm^3? Explain.

e. Do you think any object can have a smaller surface area and enclose 64 cm^3 than the box you found for question a.?

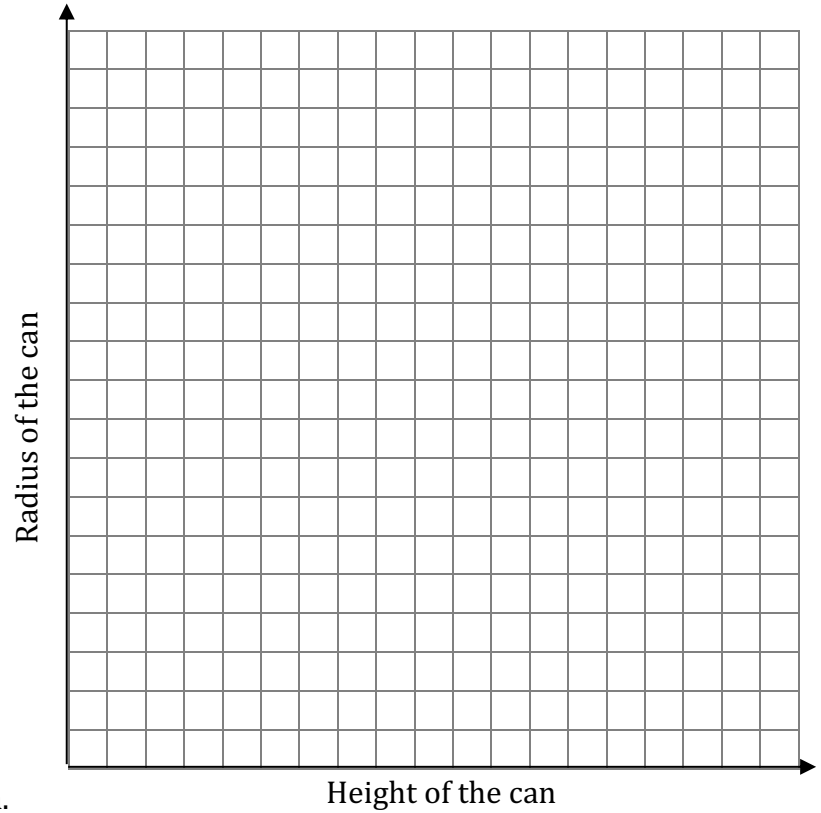
17. A room is filled with cans (cylinders). They all have a volume of 600 cm^3. Find the dimensions of at least six or more of these cans. Calculate the surface area of at least three of them.

Height	Radius	Volume	Surface area
		600	
		600	
		600	
		600	
		600	
		600	
		600	
		600	

Determine a useful scale for your axes and label them and then graph the height (x) and radius (y) of each on the graph paper below.

Describe the shape of the graph.

Calculate and add additional points to get a more complete picture of its shape.



Extra: graph radius versus surface area.