Making Math

Developing Student Mathematicians

Why CAD Should Be Part of Every Student's Math Education

Learning to work with CAD provides deep geometric learning that informs students' understandings and intuition for both applied and theoretical work in geometry. It is *quite* engaging. Students enjoy developing realistic-looking 3D designs and, because they can see immediately how something they tried worked, or not, they benefit from actionable feedback and make rapid progress. CAD offers a significant wow factor, which is then extended if it is coupled with getting to see their creations come to life in a 3D printer.

If you have not ever drafted with a CAD program yourself, set aside time to develop competence before teaching it. It is accessible but also complicated and there is a lot to learn. I had not drafted prior to creating a CAD unit and I found learning how to do so really enjoyable (even addictive, like a video game might be but isn't for me). You will probably find yourself eager to learn what more of the 2D and 3D tools do as you aspire to build more complicated forms.

As students work, they develop extensive geometry vocabulary, strong understanding of the properties of shapes, and strong visual skills. They come to understand how each dimension is comprised of an infinite stack of shapes one dimension smaller (e.g., a 3-D cylinder is made of a stack of 2-D circles). This idea relating dimensions is essential in calculus (it is at the heart of what integration is all about). They can describe different transformations (translation, rotation, reflection, and scaling) that are essential in geometry as well as in studying functions. Similar figures and measurement are common themes. It is a rich world to explore!

Teaching with CAD is in the spirit of one of our favorite dictums to start new units at an interesting middle. We are not teaching lots of vocabulary first. We are diving in and learning the language and ideas as we expand our abilities with the tool.

Getting Started With CAD: Why OnShape?

There are many CAD programs, and I have tried around a dozen of them over the years. Without question, the one that has been the best when working with middle school students or older has been OnShape. Onshape has numerous positive qualities:

- It is free. Students can sign up here: https://www.onshape.com/en/sign-up (or here for Spanish or here for French). All they need is an email (no credit card or any other information is ever requested).
- It is online, so nothing needs to be downloaded. It runs in a browser.

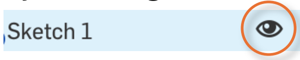
- It is accessible for students in grade 5 upward and they can develop sophisticated designs relatively quickly (a few days to good beginner results and a few weeks to more complicated designs).
- Is a powerful, professional program that they can grow with for as long as they like. I have had students continue to use OnShape for school and personal projects with an engineering, science, or art focus years after it was introduced to them in a class. It is also exciting for them to know that they are not using a kiddie version – this is a program used by designers and engineers in many fields. For artistically oriented students, it is an entirely new medium to explore. For science students, I have seen students develop new lab instruments that they needed for an experiment .
- There is always more to learn. Students who want to learn more complicated tools can move ahead with online tutorials. Once a student is logged in, they get access to many videos that demonstrate new tools and skills at https://learn.onshape.com.
- OnShape can be used to generate precise and complex forms for both 3-D printing and laser cutting. Students can see their design come to life!

Why not TinkerCad? TinkerCad, which is also free and very popular in schools, does not have meaningful 2-D sketching tools. It is closer to being a 3-D *drawing* program. It is not possible to develop relationships between shapes in the way real CAD program can, which limits the ability to modify, test, and refine solutions. The lack of 2-D tools limits possible forms and means that users don't develop an understanding of how different dimensions build upon each other. For students who are taking an engineering or mathematics class from grade 5 onward, I recommend OnShape.

Sketches

For explanations and videos about sketching, go to learn.onshape.com, click on the yellow "New to CAD" button, and then the "Introduction to Sketching" course.

Note that when working in OnShape using a mouse, standard clicks are left button clicks. For a trackpad on a Mac, it is a single click. To get the context menus, it is a right click with a mouse or a two-finger click on a trackpad.

Activity 1: Sketches are the basis for all designs. For the first CAD lesson, distribute "The OnShape layout, sketching tools, and constraints" document. Have the students follow along as you and they go through the first six steps on the first page. These steps are how they will begin all assignments. Demonstrate making a random shape or two and then encourage kids to start experimenting with each tool listed on the second page and to add any notes or observations to that page. Walk around and see what kids are trying and note any discoveries (e.g., kids asking about the implicit constraints in yellowy-orange). After a few minutes of doodling, have them click on the red X so the sketch is not saved and have them start a fresh sketch. If a student has done something they like, they can click on the green check and then start a new sketch. They can hide the first one so it is not distracting by mousing over the Sketch 1 label in the features menu on the left and clicking on the eye: 

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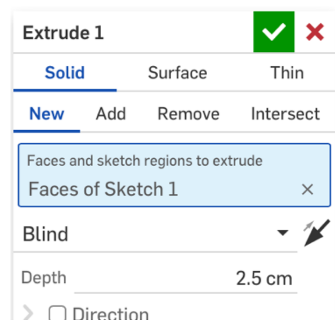
Activity 2: For the next effort, ask students to make a shape that is a single closed shape (a path that encloses a region). They can make them using the line, arc, and spline tools joining portions end to end. They can also overlap a few shapes and then remove all internal portions using the Trim tool (that looks like scissors). A picture of this process is at the top of the third page of the handout. The goal is to have a perimeter enclosing a gray region. The gray tells you that OnShape understands it as a clearly defined area.

Once everyone has a blobby bumpy sort of region, have them click on the Extrude button toward the left side of the tool menu (next to the undo and redo arrows). Extrusion is what a [Playdoh Fun Factory] does. It takes a 2-D cross-sectional shape and reproduces it moving through space perpendicular to the face of the shape to make a 3-D object (like a prism or cylinder). Geometrically, it is an infinite number of areas stacked, pancake style, to make a volume. Each cross section is congruent to the others (except when we extrude with a draft, in which case they are similar shapes, but not congruent).



When the Extrude button is pressed, a dialog box like the one at right will appear. The blue box shows what objects will be extruded. The default will be the shapes in the current sketch. Students can pick their own depth (distance for the extrusion in the depth box). Have them click the green check and then spin the view with either the Orientation Cube or by two-finger clicking and rotating the scene. They should see a realistic solid based on their sketch from all angles.

Dimensioning

Activity 3: Have students draw a rectangle. When it is begun, OnShape shows boxes for the width and height. Values can be typed in directly then, or you can make any old rectangle and then choose the all-important dimensioning tool. When in dimensioning mode, the most basic thing you can do is click on a segment and set its length. You can also click on two segments and set the angle between them. Dimensioning can control a number of other distance relationships as well: it can set the distance between two parallel segments, a segment and a point, or two points. Additionally, it can set not just the direct distance between two points, but also specifically the horizontal or vertical distance (allowing flexibility in the other direction). You choose between these options for dimensioning two points by clicking on each and then dragging your dimension up, diagonally, or sideways.

Have students experiment setting the size of a rectangle, the lengths and/or angles of a triangle, and the diagonal, horizontal, and vertical distances between the centers of two circles.

Activity 4: Note that setting the dimensions for a triangle can quickly lead to an overconstrained figure. It is great to have the class figure out when the red warnings arise. How many dimensions and of what type are just enough to fix the triangle (dragging vertices leaves the shape unchanged – it just follows along but does not change lengths or angles)? How many constraints cause a conflict? This experimentation connects directly to triangle congruence postulates and theorems that students study in geometry classes. For example, if you dimension the three sides of a triangle, then you don't get to also set an angle – the shape is now fully determined. This result is equivalent to the Side-Side-Side Triangle Congruence Theorem (often just called SSS). This discovery connects to engineering nicely and it should be pointed out that the rigidity of a triangle once you fix the lengths of the sides is why bridges and other structures are designed with triangular frames. A demonstration with Anglegs (or hard cardboard strips with holes at the end connected with paper fasteners or string) will show that three segments connected form a rigid triangle. The three lengths determine the angles. But if you attach four segments (SSSS) to make a quadrilateral, it is floppy: the angles can all shift still. For a triangle, setting two sides and the angle between them (SAS) fully determines the other three measures (the remaining side and two angles). What other combinations are just enough to determine the shape, and which leave it underdetermined or over constrained? What other shapes and number of measurements lead to a completely fixed form? For this investigation, Anglegs or their equivalent are a good complement to CAD because, depending on the order of the dimensioning and the vertex that is dragged, a figure may stay rigid when it doesn't have to be.

Activity 5: Students can practice dimensioning with the Dimensioning Practice handout. The "Dimensioning demo Video" file is for teachers and shows a couple of ways to complete the task. Be sure to watch the video in full screen so that you can see the toolbar at the top.

Constraints

For a good introductory video demonstrating inferred and directly applied constraints see: https://learn.onshape.com/learn/course/fundamentals-sketching/sketch-constraints/using-constraints. Toward the end of this video, they show how to delete a constraint. This becomes particularly helpful when an object becomes overconstrained, which means that the demands being made on it are impossible to carry out. For example, if a vertical side of a rectangle is clicked on with the horizontal constraint or two opposite sides of a rectangle are constrained to be perpendicular, then the program can't honor both requests. When one of more objects become overconstrained (and sometimes a single added constraint can throw many out of compliance), the conflicting constraints will turn red. When that happens (and it will!), just click on the undo arrow to go back a step. Alternatively, use the arrow cursor to select a specific constrain symbol in the figure and delete it.

Activity 6: Before distributing the SketchPractice activity, do a few whole class **Constraint Challenges**. If you have a video display, you can do these collaboratively, or have each student try them on their own computers. Let them know that there are missing constraints in each case and it is their job to suggest ways to fix the situation.

Challenge 1: A Leaning Triangle

- Sketch an obtuse triangle with a horizontal base. Tell the class it is yearning to point straight up.
- *What constraint tools can help it achieve this goal? How can you use the constraint tools to make sure it is always upright even as vertices are dragged about?*
- Notes: One option is using the equal tool for the non-base sides forcing it to be isosceles. Another is to construct the midpoint of the base and then a perpendicular line through that point. Then, they can use the coincident tool to force the top vertex onto that line. Note, making the base angles equal is not possible with a constraint tool, but the angles can be dimensioned to an equal value.

Challenge 2: A Wobbly Square

- Sketch a random quadrilateral with the line tool and make sure it has no distinctive properties. Tell them that it is, at its heart, a square and just needs their help to make sure that is the case.
- *What constraint tools are needed to help it become a square? How can you use the constraint tools to make sure it is always a square even as vertices are dragged about?*
- Notes: Perpendicular and equals tools are one approach and you can point out that these choices mean it can still be dragged into differently sized squares (while dimensioning the figure would have chosen a specific square). If the changes are looked at one step at a time, students can be prompted to observe that they don't need to make all four side pairs perpendicular. How many do they need and why? If they do perpendicular first, how many equal side pairs do they need (only 1)? Why? If the equal constraint is used first and then the perpendicular constraint, these answers flip. These questions can be framed as "what are the fewest constraints that we need?" Less obvious ways to make the figure a square are possible. Students can introduce the diagonals as braces and make them equal and perpendicular with coincident midpoints (which, admittedly, is unlikely to come up in class unless you demonstrate it just to show that many approaches are possible).

Challenge 3: Three Circles Challenge

- Start with three random circles anywhere in the sketch and end up with them all the same size and equally spaced.
- *What are two different ways to do this? What is the fewest number of constraints you need?*

Challenge 4: Mystery Constraints

- Show a finished sketch that you used constraints to make, such as a simple logo or other design with some symmetries or other features. The constraints are normally hidden.
- Ask students to write down at least two or three constraints they think you used to make the figure. Note that there are different ways to achieve the same result.

- Once they have their answers, you can reveal your choices by mousing over a point or segment. When you do, OnShape will show the different constraints impacting the object and if you mouse over one of the constraint symbols that appear, it will highlight the figures involved in that particular relationship. For example, if you mouse over a segment and the perpendicular symbol appears and you mouse over that symbol, OnShape will highlight the two segments that are perpendicular.
- Once a shape is well understood, you can delete a constraint (by two-finger or left-clicking on the constraint icon that arises when you mouse over and choosing the last entry in the menu "Delete sketch entity"). Then demonstrate what aspects of the shape are no longer preserved (try dragging different points).
- After showing constrained and under-constrained examples, ask the students for a constrain that will conflict with current ones and cause an overconstrained warning. (e.g., two segments can't be both perpendicular and parallel or, more subtly, if you construct a right triangle, you can't force the hypotenuse to be equal to another side).

Activity 7: Distribute the "Sketch and constraint practice" handout and give them time to work. Students who work more quickly can be directed to try some of the harder shapes. As you move about the room, remind students to check constructions every two or three steps by dragging vertices.