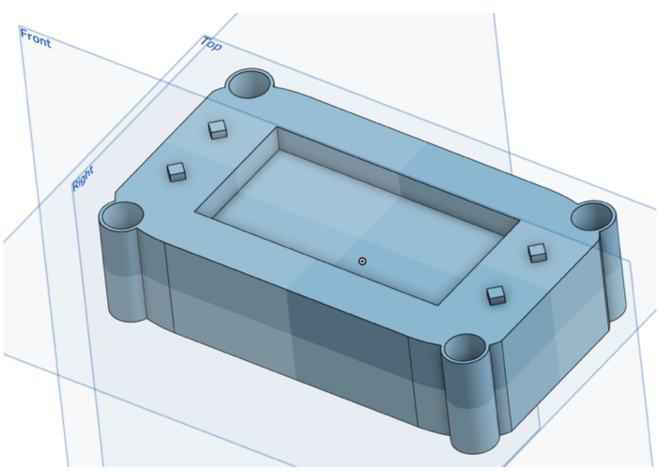
Making Math

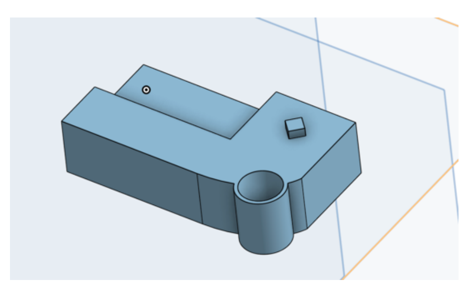
Developing Student Mathematicians

Symmetry

An object has the property of **symmetry** if there is a transformation (motion or mapping) that leaves the object unchanged (it is the same shape in the same place). For example, in the graphic of names at right by puzzle maker, artist, and amateur mathematician Scott Kim, all of the names have either rotational symmetry (a 180° spin around their center lands them back where they began, as with **annie**), reflection symmetry (over either a horizontal – **DEBBIE** – or vertical line – **TIMOTHY** – through their center), or both (e.g., **O++O (Otto)** has rotational and both types of reflection symmetry).

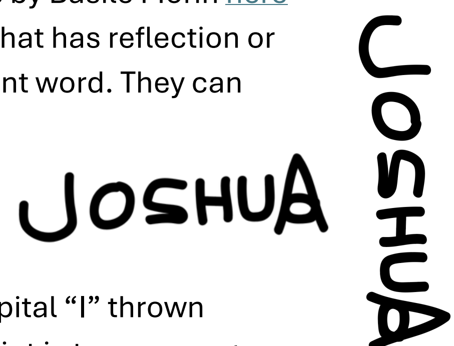
Recognizing the symmetries in an object makes it possible to more efficiently develop a model of that object. However, while students may see symmetry, they may not be familiar with its actual geometric properties or how to take advantage of those properties in identifying the basic subunit that can be replicated to make an object. The images below show a shape that has reflection symmetry in all three orthogonal directions (over the Front, Top, and Right planes). The subtle differences in the shades of blue show you that each octant of the shape are reflections of each other and the whole shape can be made from the single form at right. That subunit was extruded from parts of the same 2-dimensional sketch and then the 3-D mirror tool was used to build up the final object in three steps using each of the planes once. Alternatively, the flat surfaces hidden below and to the left and right in back of the subunit itself can be used for the 3-D mirror tool.



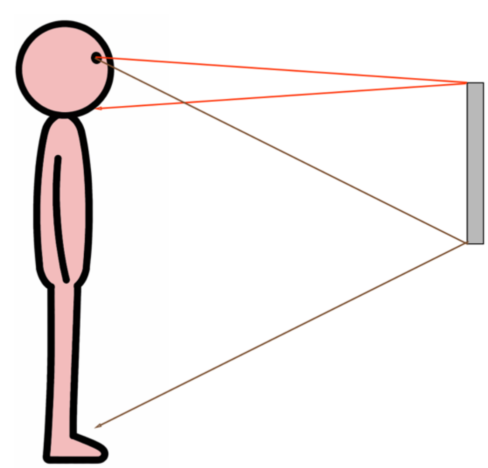


Activity 1: Before launching into symmetry tools in OnShape, it is worth spending some time with more hands-on activities that expand students' ability to mentally manipulate and transform images. Share the above definition of symmetry and names graphic with students. Kim called his designs *ambigrams*, where a word or words read as a different word when rotated or as the same word but with different letters represented by the same rotated or reflected shape. As an example, in the ambigram of "ambigram" below, the "b" and "g" are the same shape and the "am" rotates but still reads as "am". For standard fonts, "am" rotated 180° looks like "ɯɐ" (click here to see an animation of the rotation, or just print it out and spin the page).

Also share with students the wonderful collection of ambigrams by Basile Morin here and ask them to come up with a word (not necessarily a name) that has reflection or rotation symmetry or that reflects or rotates to become a different word. They can experiment with capital and lower-case letters and look at different fonts and some of the ways Kim and Morin match different letters cleverly. At right is my very quick personal example (humor me, please), in which my first name rotated 90° becomes a math term ("conic" read top to bottom with a capital "I" thrown in and a matching diagram of a conic section at the end). The point is to encourage students to play with shapes and transformations and to be creative. Some more artistically oriented students will have fun developing more appealing efforts than this one (which you should not share with them in order to preserve my dignity :--).



Activity 2: Mirrors are, not surprisingly, a useful tool when teaching about reflections. Hand out the Mirror Activities problems, pieces of blank (copy) paper, and a small plastic mirror to each student and also pair students up to discuss their solutions and, for the two mirror problems, to work together. Afterward, you may want to share a question a student of mine explored, which is how much can you see in a mirror and why? Taking a mirror *H* cm high, how much of *your own image* vertically can you see of yourself? At right is a diagram of the situation showing that our person can see from their chin (such as they have one) to their ankles. How much is that relative to the height of the mirror? What is the person moves back further or closer to the mirror? Kids can work out this image for themselves by exploring different vertical mirror placements and mirror sizes (by combining their small



mirrors). Note: the vertical view is cleaner than horizontal ones because our eyes are in different horizontal positions.

Activity 3: It is time to get more technical about reflections. Have students draw a line *L* on a blank page and a single labeled point *P* somewhere on the page. If a student puts the point on the line, that is fine, as it will give the class the opportunity to highlight that the image of a point on a line of reflection is just the same point. If you have square patty paper (which is translucent), that is even better for this activity. Have students swap their pages with a partner who should, as carefully as they can, mark where the reflection of point *P* over the line would land. They should label this *image* point P'. The partners should then swap back their pages and discuss whether the reflections look accurate. How can they check their estimates? What guidance did they follow in trying to locate the right spot? Students should share that they were looking for the same distance away. How did they gauge the right direction? A student may or may not be able to convey that they were moving “directly” toward the line or in some way capture the notion of moving at a right angle to the line. Without clarifying further, have the students fold their paper at the reflecting line. Roam the room to make sure they are folding on their lines. They can now hold it up to the light to see how close the original and image points are (if using regular paper, black markers will make it easier to see the images). For a perfect reflection, they should land on each other. Have them mark on their page where the original point appeared when the paper was folded to get an accurate image point. Ask them to use a straightedge and draw the segment *PP*', which is written symbolically $\overline{PP'}$, connecting the original and image points and then seek comments about the relationship between the reflecting line and this segment. Turn their comments into something along the lines of this definition: **the reflection of a point *P* over a line *L* is the image point *P*' if and only if *L* is perpendicular to $\overline{PP'}$ and L contains the midpoint of $\overline{PP'}$**. Or, the final portion can be and ***L* bisects $\overline{PP'}$**.

As students work on the activity below, remind them how to use this definition to check their work. Are they moving perpendicularly across the reflecting line and equidistant from it?

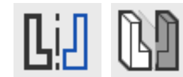
Activity 4: Have students work on some or all of the Seeing Shapes reflection problems over more than one in-class or at-home sessions. Note that Exercise 6-C, which incorporates oblique reflection lines, is much harder for students than horizontal or vertical lines. They can try rotating their page so that the lines are not diagonal at first and then see if they can make the transition to working over angled lines.

If you are looking for more hands-on activities, there is an entire book with rich geometry connections based on reflection: *Patty Paper Geometry* by Michael Serra (with inexpensive used copies available).

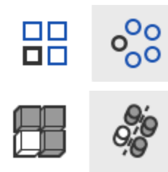
Activity 5: And now we return to CAD. The OnShape tools that involve reflections and symmetry are:

- The **symmetric constraint**, which forces two pre-existing shapes in a sketch to match their distance or size. See sample videos here and here.

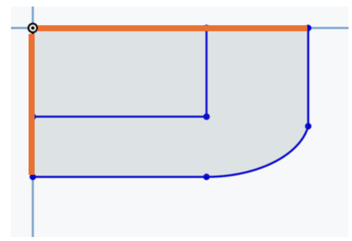
- **Mirror tools**, which take a 2-D figure or a solid and duplicate them over a mirror line or plane. Here are sample videos for 2-D and 3-D mirror tools.



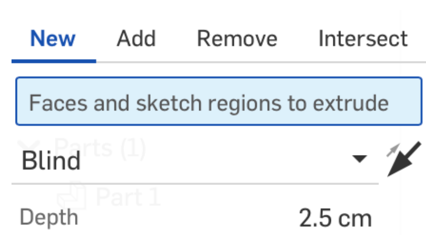
- The **linear** and **circular pattern tools**, which produce symmetrical reptitive designs efficiently within both sketches and the 3-D mode.



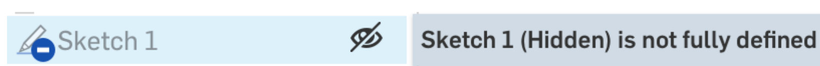
To preview this activity, watch the short *Multiple reflections demonstration* video, which also shows how to make several different extrusions from a single sketch.



a) Have students start a sketch in one of the planes and begin by drawing segments from the origin along two of the axes. They should then add additional elements so that there are two or more closed regions in a single quadrant of the plane.
b) Have them extrude a shape but not the entire sketch in one extrusion.
c) Students can click on the different regions to unselect and select different ones. When the blue box in the Extrude window says “Face of Sketch 1” instead of “Faces…”, they can click the green ok check mark to complete the extrusion.



d) After their first extrusion, they can then make sure the sketch is still visible by clicking on the “eye” icon at the right of the sketch entry in the Features panel on the left.



e) Now have them start another extrusion based on other parts of the sketch not used in the first extrusion. They should be sure to pick a different extrusion depth (or the different extrusions won’t be distinguishable).
f) Once all regions of the original sketch have been extruded, students can use the 3-D mirroring tool, as demonstrated in the video, to first double, then quadruple, then octuple the original part over the Top, Front, and Right planes (or over the three flat surfaces in the part that are in those three planes).