Mathematical Modeling: Teaching the Open-ended Application of Mathematics

Chapter 0-1
Introduction

Joshua Paul Abrams
www.makingmath.org

To my parents – wonderful models for how to live a life.
To my high school teachers Stewart Galanor and Antonia Stone
who taught me well and then put me to work.
To Henry Pollak for inspiring so many of us to teach modeling.
To all of my creative, thoughtful, and good-natured students.
To my wife, Emily, for all of her encouragement and get-up-and-go.

Using This Curriculum

We are excited to share these materials with teachers to adopt and adapt as you see fit. This modeling curriculum is flexible. At some schools, it has served as a full-year course for high school juniors and seniors that was a graduation requirement within the mathematics sequence. Alternatively (or additionally!) different skills and explorations can be introduced throughout middle and high school, and the teaching notes for each unit will suggest ways that these activities and projects can be woven into classes from grade 6 through 12 as a modeling strand.

This curriculum provides a framework for thinking about the teaching and learning of mathematical modeling as well as in-depth descriptions of many class activities and project assignments. In some cases, more activities are provided for a given topic than would be used in a given class in a given year. Many topics, such as numeracy, function models (here and here), probabilistic reasoning and simulations, and ranking functions, benefit from repeated use at ever-greater sophistication throughout the years.

We believe the most effective courses reflect teachers' and students' own intellectual passions. We hope you will borrow and blend what is presented here with your own best practice. Folk singer Woody Guthrie once claimed that "plagiarism is the basis of all culture" and criticized a fellow folk singer by noting that "he only steals from me, but I steal from everybody." In that creative spirit, we offer not a single, "teacher-proof" package, but a teacher-dependent resource. If you develop or improve a lesson, please feel free to share it with Making Math and we can share it on the website.

What is Mathematical Modeling?

Mathematical modeling is the process of using mathematics to study a question from outside the field of mathematics. A mathematical model is a representation of a particular phenomenon using structures such as graphs, equations, or algorithms. This course gives students practice formulating interesting questions from fields such as science, entertainment, politics, or design. It teaches the specific skills used in creating and interpreting mathematical models. These models, in turn, produce new understandings about the original settings of interest and help students answer the questions that they have posed.

The culminating activity of this curriculum (as described in the final unit) is a several-weeks-long group project for which students pose their own questions and develop an original model. Sample questions that students have explored include:

How does accumulated dirt affect the melting of snow mounds?

What harvesting guidelines will protect a lobster population in the face of uncertain population size and unknown natural variations in the environment?

How can delegates to the United Nations be seated to maximize harmony?

How should a town structure the penalties for speeding tickets in order to generate the greatest revenues?

What arrangement of ceiling lights provides the most even illumination of a room?

As these questions suggest, the goals for a modeling endeavor are varied. Modelers seek to gain understanding, predict outcomes, make decisions, and develop designs. The snow group, inspired by the ever-dirtier mounds piling up during a snowy winter, simply wanted to understand what they were observing. They knew that the process of identifying variables, creating representations, working with those abstractions to generate new information, and determining the significance of that information to the original question would help them toward that end. The lobster group worried that harvesting regulations might be set to leave the bare minimum number of lobsters needed to replenish the stock. They developed a model for predicting population changes over time. Using the model, they discovered that a too aggressive policy would result in a complete collapse of the population, if a decreased birthrate or other perturbation reduced the stocks below the minimum level. The U.N. and speeding ticket groups both sought decisions that would optimize some variable. Optimization is the most common goal for student questions. The illumination group was interested in a design that would improve their immediate working environment (their classroom was a windowless basement room which made lighting a major concern for these cave-dwelling students).

Why Teach Mathematical Modeling

This course was designed to achieve several objectives:

- Mathematics classes should have a coherent focus that unites the skills and ideas being studied so that students can use their learning in meaningful ways *in the present* and as they move forward with their schooling and adult lives.
- Academic subjects provide ways of knowing (criteria for how we evaluate the truth of claims) and questioning. They give us tools and perspectives for understanding our world as well as past, future, and hypothetical worlds. For

example, in trying to understand the industrial revolution, we can explore it scientifically, mathematically, literarily, and in its historical context. Each of these lenses helps us to understand a question more completely. Students should not just be taught officially accepted facts and specific skills – they should be put in the game as junior mathematicians, writers, scientists, historians, and more asking their own questions and applying the disciplines to glean new insights into those questions.

- For mathematics to be a tool that students use to understand their world, they must be able to approach unfamiliar problems, to determine which skills they have that might be relevant to the problem, and to apply those skills to come up with new knowledge. Because mathematics is a vast and always-growing discipline, and because the range of mathematical ideas that might serve in studying a real-world problem is broad, we cannot teach our students all of the technical content they will need to know. Thus, our greatest challenge is to teach them processes that they can use to connect all areas of math to questions from outside of the field of mathematics. Then, as they learn additional mathematics skills, these can be incorporated into an ever more flexible and powerful toolkit for pursuing their curiosity.
- Trying to understand our world is almost always an interdisciplinary effort. Mathematical modeling is about the ways we connect abstract mathematical thinking to concrete situations. Some of a student's learning must cross the permeable boundaries of the different disciplines and foster an understanding of the capabilities and uncertainties inherent in how each discipline informs us. Using math in these rich and varied ways provides an added benefit in developing learning that is *connected* and *memorable*. Students who just learn skills by practicing them in predictable ways (worksheets and tests where they know that skill will be required) may "know" the skill, but it is inert for them – they don't know when to use it without prompting. Life does not come with instructions! When students use math in novel ways, they are developing connections between their skills that enable them to be more readily retrieved from memory and applied in new circumstances. Educator and philosopher John Dewey emphasized that we don't learn the basics by just repeating them, but by engaging in rich tasks that require them. These skills are then a part of a meaningful web of ideas. This increased meaningfulness and the

engagement that comes with this kind of learning makes for longer-term retention of skills and information.

- Studying mathematical modeling provides productive challenge for all students from those who think they don't like math to those that jump in eagerly. One of the wonderful aspects of this work is that there is not just one right answer. The questions are too complex to provide that kind of simple result. There isn't even only one right question! Good work is about making decisions and justifying them thoughtfully and being clear about the strengths and gaps in a model and the limits of the information generated. Engaging in these investigations will immerse your students in the subtlety, breadth, depth, utility, and beauty of mathematics, and train them to be able to use mathematics in new and fruitful ways. All students completing this curriculum are prepared to study many mathematics, science, and social science subjects with a new set of tools and perspectives at their disposal.
- There is a constant interplay between pure and applied mathematics questions. Students should see cases in which whole fields of theoretical mathematics arose from real-world questions that necessitated their invention. In the twentieth century, the development of first telephone and then computer networks led to new mathematical subjects and ideas in graph theory, statistics, and data representation, compression, and error-free transmission. Many of these ideas, such as minimal spanning trees and error-detecting codes, are accessible, in their least complicated form, at even the middle school level. As telecommunications continue to grow, cellular networks continue to require new mathematics tools to make service reliable and efficient.
- We want students to be excited about their learning. Mathematical modeling engages them because the topics are sometimes surprising and playful and often address themes about which they care. They get to connect the work to their own interests. This kind of intrinsic motivation is essential to effective learning.
- Too many students reach college mathematics classes as rigid thinkers with underdeveloped intuitions about quantities and functions and with little persistence or familiarity with experimentation as problem solvers. What we dedicate class time to is what students will master. Mathematics classes must devote a significant portion of their time and activities to the higher order

habits of mind that produce rigorous reasoning, creativity, and flexibility. These will not materialize out of long-term exposure to studying and practicing technical skills alone. The activities in this curriculum are created to nurture careful, critical analysis, original problem posing, and the patience essential to success with challenging endeavors. In the Third International Math and Science Study (TIMSS), American mathematics curricula were criticized for being "a mile wide and an inch deep." Our course introduces fewer mathematics topics, in depth, with time for explorations and projects. Short run-ups only lead to short jumps; long run-ups lead to big intellectual jumps.

- Finally, and most importantly, in a vital democracy, a primary goal of schooling should be the development of thoughtful, informed, and active citizens. Mathematics is an indispensable tool for reaching this goal. With mathematics, we can ask and answer important social, scientific, and political questions and analyze the claims that policy makers present to us. Mathematical modeling skills and attitudes will help our students become more questioning and more curious and, therefore, less passive and less gullible receivers of knowledge. Toward that end, students completing a modeling course or sequence should be 1) informed, critical consumers of publicly disseminated mathematical and statistical claims and 2) able to take an issue of interest and use mathematics to better understand it, to discover new facts about it, or to generate new questions for thinking about the matter. They should not be forced to merely accept things the way they are but be able to imagine the way our world might be and assess the impact of those differences. Most often, public discourse about the importance of mathematics education emphasizes preparing students for a career and the significance of this preparation for the future of our economy. Students who have taken our modeling course have returned to report how much it has helped them in their work in science, business, and industry. Both the political and personal contributions of being able to interrogate our world through the lens of mathematics are essential and should receive equal attention.