***RACE TRACK*: A CARTESIAN STRATEGY GAME**

**Welcome to Race Track! This game is a simulation of race cars that you can play on graph paper.**

**Object of the Game**: To reach the Finish Line before your opponents do (i.e. in the fewest moves).

**Set-up for the Game**: On a piece of graph paper, draw a curved race track several squares wide with straight, horizontal or vertical, Start and Finish lines (see diagram 1). Be sure that the track does not get too narrow.

Finish

Start

**Rules of the Game**: Each player drives a car (a “point car”, really). The cars travel along the corners of the squares (the lattice points) of the graph paper (the points where the lines intersect). Each player chooses a distinct point on the Start line. Cars move according to their velocity which starts at (0, 0) for each player. The velocity is represented by two numbers which tell how fast the car is moving sideways and up or down each move: (*x* or horizontal change, *y* or vertical change). So (0, 0) means that the cars are not moving at the beginning. A velocity of (4, -2) means that a car moved to the right 4 units and down 2 units. You steer your car by changing (accelerating or decelerating) these two numbers. Just as a fast-moving car takes time to stop or change direction, your velocity can also change only gradually. You may alter your horizontal and vertical motion by no more than 1 unit per turn. For example, if on your last turn you moved 3 units to the right and 4 units up – your velocity was (3, 4) – then on your next turn you must move 2, 3, or 4 units to the right and 3, 4, or 5 units up. That is, you may slowdown (–1), maintain (0), or increase (+1) your speed for each direction (see diagram 2).

**Diagram 1. A Sample Course**

(2,5) (3,5) (4,5)

(2,4)

(2,3)

(4,4)

(4,3)

(3,4)

(2,5) (3,5) (4,5)

(2,4)

(2,3)

(4,4)

(4,3)

(3,4)

# Diagram 1. A Sample Course

**Diagram 2. The 9 possible moves following a move of (3,4)**

**Diagram 2. The 9 possible moves following a move of (3,4)**

Note that a move of (1, 5) and (-1, -5) are not the same. The first is a move 5 up and a little to the right. The second is a move 5 down and a little to the left (see diagram 3). In general, if it looks like your car made a sudden change in direction, you probably made a mistake moving (unless it slowed down first, in which case a sharp corner is possible).

y

x

**Diagram 3. Opposite directions have opposite sign**

The following additional rules should be considered:

1) Any player whose car leaves the track is out of the game. Each move must be connected by a straight segment. If the segment connecting the points touches or crosses the boundary of the track, you are similarly eliminated from the game (you crashed into a wall).

2) Cars may occupy the same spot at the same time. If students mimic each other’s moves too much, you can reverse this rule.

3) All players must be allowed the same number of moves. If two players cross the Finish line on the same turn, the one who is the greatest number of squares past the Finish line wins.

**C r a s h !**

 **Sam Karen**

**W i n s !**

Sam Karen

(1, 1) (0, 1)

(1, 2) (0, 2)

(0, 3) (0, 3)

(0, 4) (-1, 3)

(-1, 4) (-2, 3)

(-2, 3) (-2, 2)

Crash! (-2, 1)

(-2, 0)

(-3, -1)

Wins!

Diagram 4 shows a sample game. Look at the directions of the players’ moves and the sign of each number. Notice that no number changes by more than 1 unit per move. Sam was unable to avoid crashing because after going +4 vertically on the previous turn, he had to move at least 3 units up on the next turn and therefore could not avoid the wall.

**Possible modifications of the game**: Once you have had some experience with the basic game, you might want to try these variations: Oil Slicks — draw regions in the middle of the track which require the driver to keep their velocity unchanged for any moves which even partly overlap the region. Mud — regions which require the driver to slow down both horizontally and vertically (x – 1, y – 1) for each turn wholly or partly overlapping the region. Croquet — instead of the track, draw numbered “hoops” (segments) around the page which must be driven through in sequence. Invent your own modifications and explore their consequences!

**Diagram 4. A Sample Game**



**F I N I S H**

|  |  |  |
| --- | --- | --- |
| **Turn #** | **Player #1** | **Player #2** |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |

**START**



|  |
| --- |
| **S** |
| **T** |
| **A** |
| **R** |
| **T** |

(x, y) y

x

|  |
| --- |
| **F** |
| **I** |
| **N** |
| **I** |
| **S** |
| **H** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Turn | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Player #1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Player #2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Turn | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Player #1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Player #2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

(x, y) y

x

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Turn | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Player #1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Player #2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Turn | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Player #1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Player #2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |