Making Math

Developing Student Mathematicians

Mind Reading

This class activity introduces the power of expressions to represent infinitely many problems all at once. It is in the spirit of one of our favorite dictums to start new units at an interesting middle.

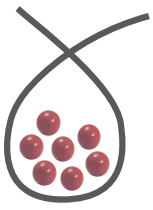
Tell the class that you are going to read their mind. Ask them to write down a number between 1 and 20 {any number actually works, but you want them to be able to carry out the calculations efficiently and accurately}. Then ask them to do each of these steps and to write down each result as they go:

- Add 8 to your number
- Multiply that sum by 3
- Subtract 12 from that product
- Double your last result
- Divide that by 6
- Subtract your original number
- Now, with fanfare, announce that their result is 4. Alternatively, if you want to be really showy, you can, prior to class, put a sticky note under each desk with a 4 on it and now tell them to look for your guess for each of them, so it seems you have individualized your guesses.

Now ask students how you knew. Typically, they will start to see that there are steps that undo each other. If no clarity emerges, you can ask them to do a second example side by side with the first and see if the comparison is enlightening.

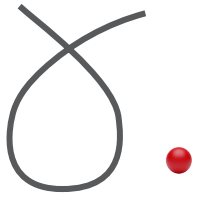
Whether or not a complete explanation emerges, suggest that the class carry out the steps for all possible starting points at once using a bag that contains as many marbles as their initial guess. Here is a bag with the starting value 7 and then the one we are going to use. The second one is not empty, it is just opaque, so we can imagine that each student has the correct number of marbles matching their first guess in it.

x-ray view of a bag with 7 marbles

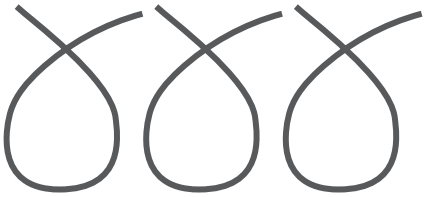


a bag for all possibilities

Now work through the steps again asking for what the drawing should look like at each step. Explain that constants can be represented with marbles outside if the bag (since the bag's contents are different for different students). So the original number plus one would look like this:

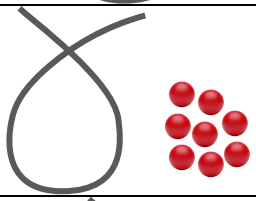
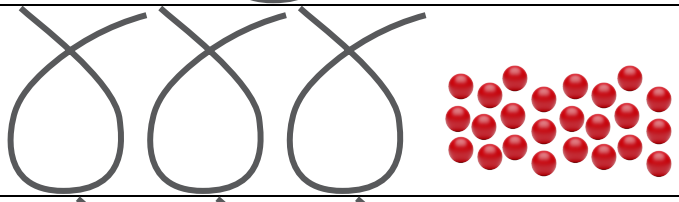
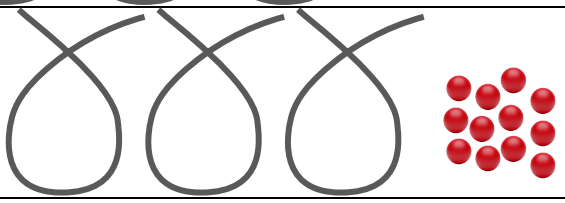


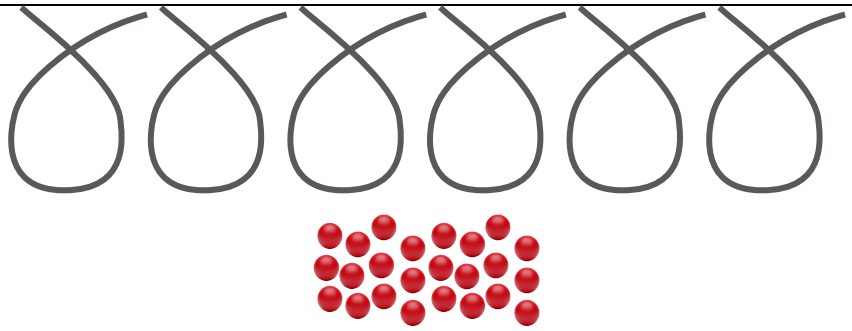
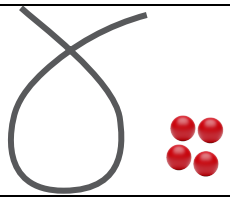
And three times their original number would look like this:



It is important to note that while the bags can contain different numbers of marble for each of their situations, they all contain the same unknown number of marbles within each case. So, if you started with 9, all bags in your diagrams will have 9 marbles in each for all steps and mine will have some other value.

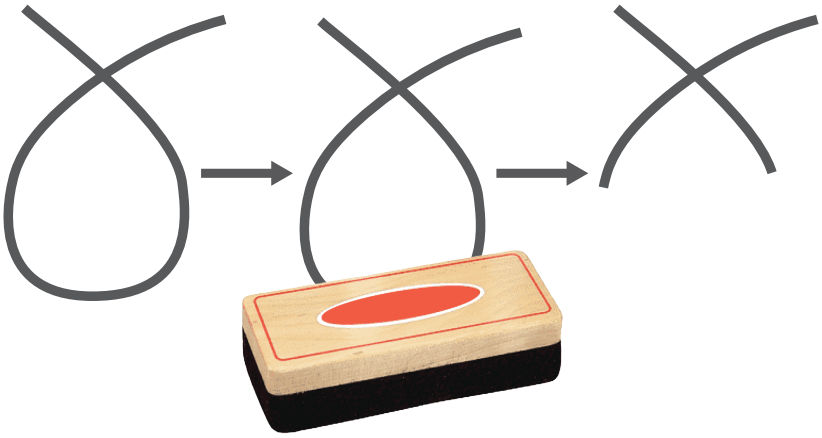
Here we go:

Pick a number	
Add 8 to your number	
Multiply that sum by 3	
Subtract 12 from that product	

Double your last result	
Divide that by 6	
Subtract your original number	

This sequence shows that whatever a student began with, there is no bag left at the end, so their choice did not matter. 4 is the invariant result!

At this juncture, my teacher, Stewart Galanor liked to grab an eraser and do this to each bag:



He wanted the students to picture the bag whenever they saw a variable, such as *x*. It is worth reinforcing this image regularly.

Students can now be asked to invent their own mind-reading/magic trick and to demonstrate with bag and marble diagrams how it works. After that, they can be asked to add a third column that shows the less graphic demonstration, which, for the above problem, would be:

$$X \rightarrow X + 8 \rightarrow 3X + 24 \rightarrow 3X + 12 \rightarrow 6X + 24 \rightarrow X + 4 \rightarrow 4$$

Explain to students that 3*X* means 3 times *X*, but also allow, and even encourage, them to write 3**X* until the implied multiplication is needed and helpful.