Patterns Rule!

Exploring Iterated Patterns

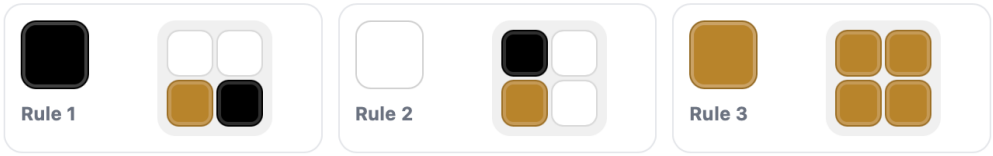
Patterns Rule! generates **substitution patterns**, which are built by repeatedly replacing each colored square in a pattern with a smaller grid of squares according to a set of rules.

- Each **rule** tells you how a single square of a given color should be replaced.
- All squares are replaced **simultaneously** at each stage.
- Repeating this process creates increasingly detailed patterns.

How the process works

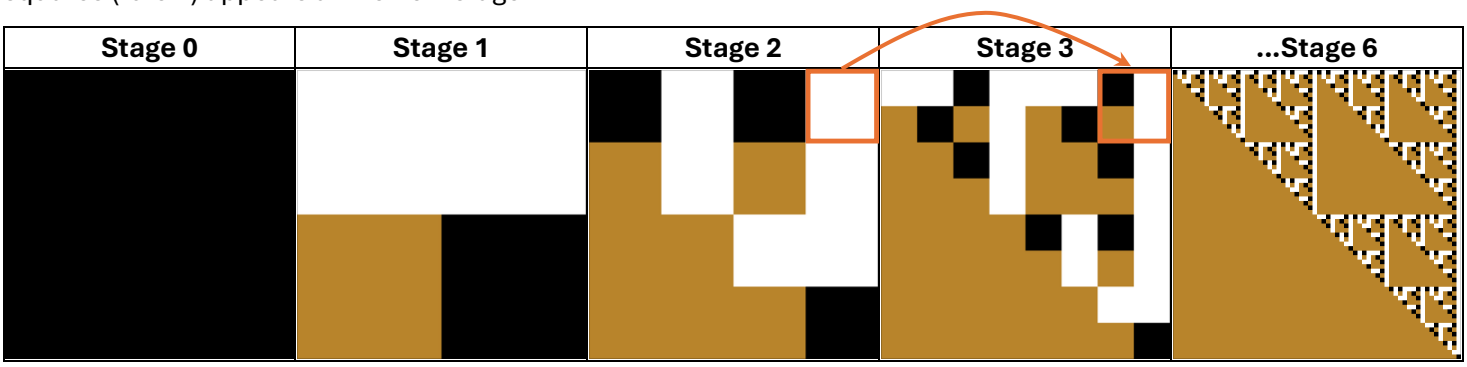
1. **Start at Stage 0**
   The pattern begins as a single square using the first rule's color.
2. **Apply substitution rules**
   Each square is replaced by a small grid (either 2×2 or 3×3) based on its rule.
3. **Advance stages**
   At each new stage, *every* square is replaced again using the same rules.
4. **Track proportions**
   As the pattern grows, the program tracks how much of the total area belongs to each color.

Here is an example using these three rules for black, white, and brown squares:



Rule 1 means that all black squares should be replaced, during the subsequent stage, by four squares (each half as wide and half as high as the original square, so that they take up the same area): two white squares on top and a brown and black one below. The third rule assures that all brown squares remain brown (they are not replaced by any other color).

Below are some stages for this replacement pattern. In the orange squares, you can see how the pattern for white squares (rule 2) appears at the next stage.



Main controls and features

# of Rules

Pick from 2 to 6 rules.

Grid Size

Choose 2x2 or 3x3 grids.

Stage Slider

- Move through the substitution stages.
- Higher stages show more squares.
- The graph and percentages update with each stage.

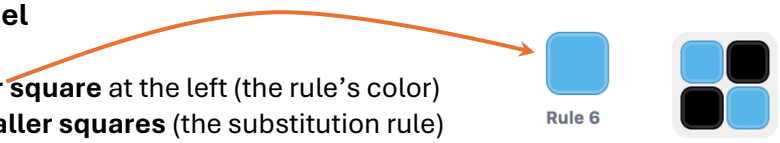
Grid

Superimpose a grid of outlines to see the substitutions better.

Pattern Rules Panel

Each rule has:

- A **large color square** at the left (the rule's color)
- A **grid of smaller squares** (the substitution rule)



Click any small square to change which rule color replaces that position.
Click any color square (the large square) to change a rule's color using a color picker.
Changing a color updates the pattern and the graph.
The default colors are chosen to remain distinguishable and accessible for users with different types of color blindness.

Rule Buttons

- **Solid rules**
  Sets each rule to a uniform grid of its own color.
- **Randomize**
  Randomly assigns each small square to one of the available rule colors.

Graph and Summary

The graph shows how the **percentage of each rule color** changes across stages.
Above the graph:

- Each rule's **percentage** – how much of that color fills the pattern at the current stage – is shown.
- A **simplified fraction** shows the same value as a part of the total area (for example, 12.5% → 1/8).

Save, Open, and Copy

- **Save** – lets you name a file and download it, so you can revisit, revise, or share it. Saved files are in a format (.json) that does not directly tie to the program, so they should be reloaded using the Open button in the upper right corner of the *Patterns Rule!* window.
- **Open** – loads a previously saved pattern file (.json).
- **Copy Pattern** and **Copy Rules** – depending on the settings of your browser, these buttons will copy an image of the main pattern or the rules to your clipboard or will download the image to your Downloads folder in .png format.

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