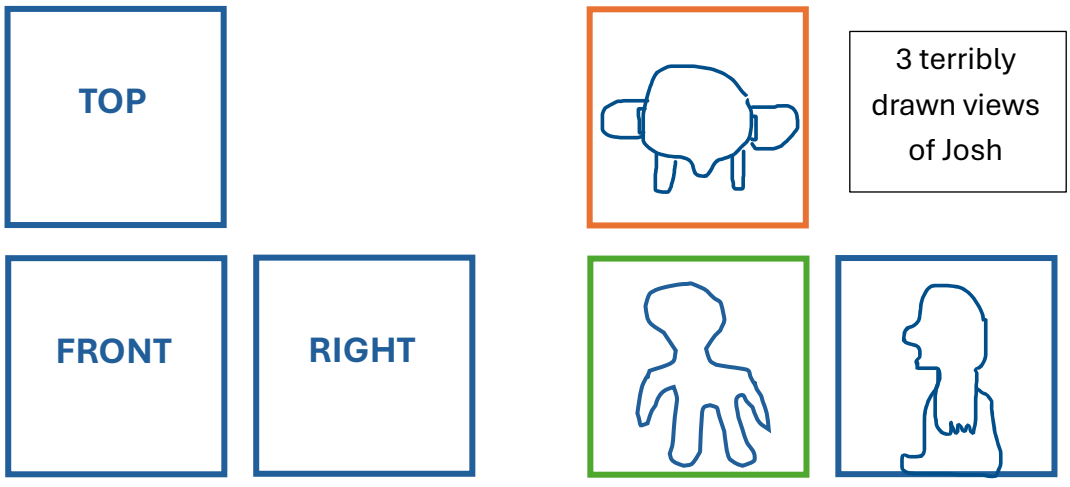
Making Math

Developing Student Mathematicians

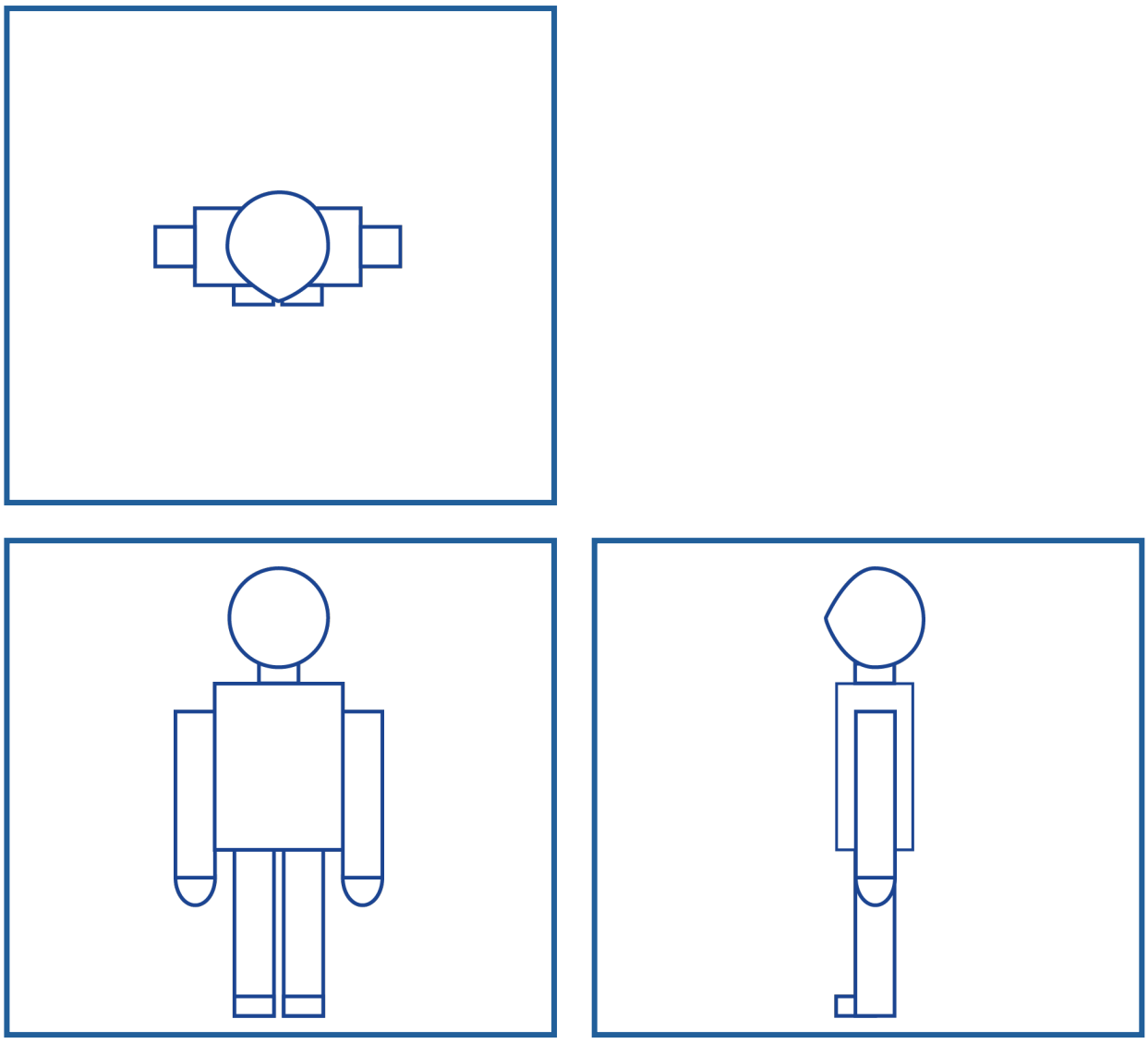
Orthographic projections

Orthographic projections take a 3D object and use parallel projections onto a plane to convey what the shape looks like from different angles. All sight lines are parallel (not perspective lines that converge) to make 2D drawings. This video demonstrates the concept, although it calls what our materials call the front view the left view and what we mark as the right view the front view (that is, their perspective is rotated 90° from these materials).

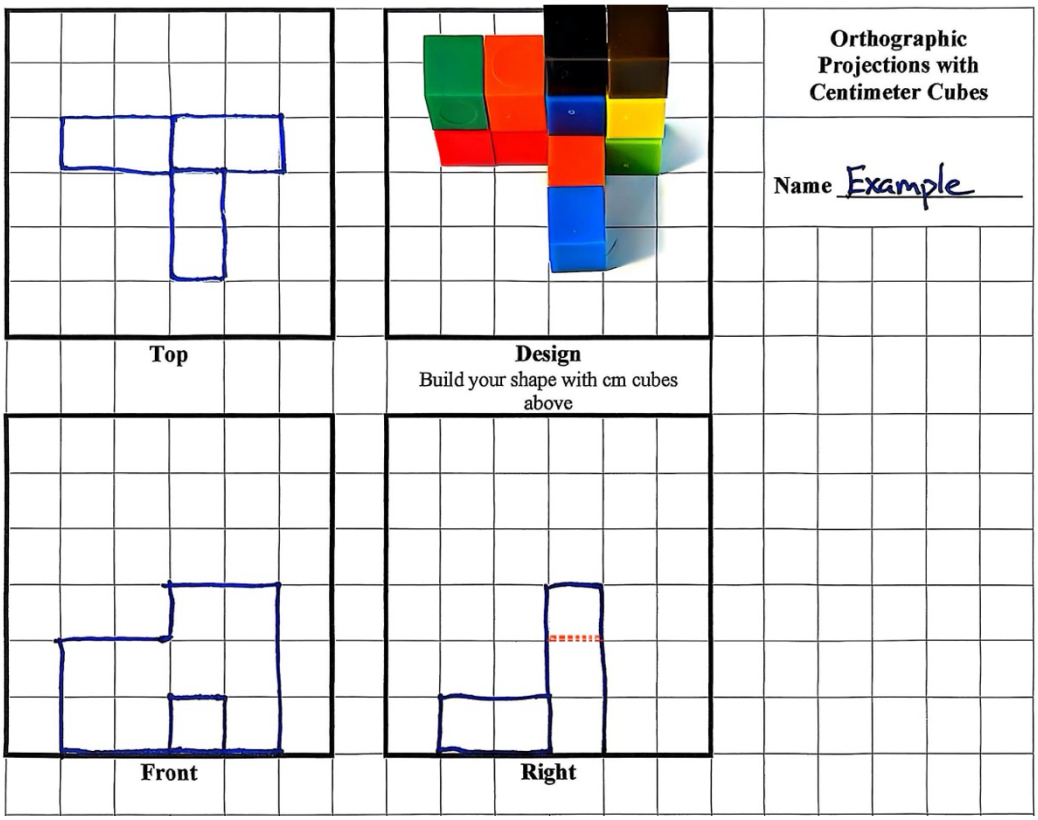
When you introduce the idea in class, begin without the formal language and ask students to imagine that they are hovering above you (the teacher) and to do a very quick outline sketch of what they would see from a bird's eye perspective. I am notorious in my classes for my bad drawings and offer up something like what you see in the orange box below: a head with a nose pointing downward, two narrow ears, shoulders sticking out to the left and right, and feet poking out on either side of my nose. It's okay if you don't see it! The students do a better job. Then ask them for the outline of what they see facing my front (see the green box for my interpretation that definitely looks less like a human and more like a four-limbed octopus). Lastly ask them to sketch you in profile (turn to your right to show them your left side – this is the Right view, because it is the same as them moving to the right to see you from the side).



On the next page is a more stylized but clearer version of what the three views of a standing person might look like. For many shapes, especially those with symmetry, these three views will suffice. For example, two side views may be identical, but students may note that that is not always the case (the bottom view of a person's feet will look different than the top view of their head). The bottom, left, and back views of shapes can be added when they think they are helpful (and there is extra room on the handouts if they so choose).

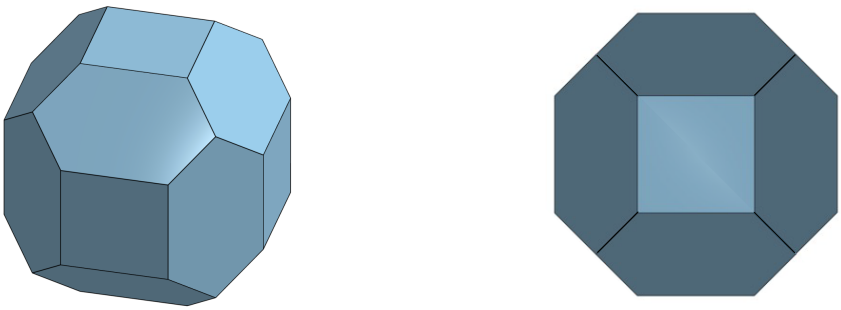


After introducing the concept of orthographic projections, Distribute a dozen or more cubes to each student and a copy of the Orthographic Projections with Centimeter Cubes handout. Ask them to place some cubes in the Design box stacked as they wish but staying within the grid. Then have them really get down at eye level with their design and sketch the outline of each of the three views (front, top, looking from the right). Note that any faces of the cubes that are adjacent will appear as one connected figure (see example handout below). The only time an internal edge is drawn is if there is a change in distance from the viewer (a cube is closer or farther away). These views do not show what is going on behind the visible surface: in the example below, a dashed orange segment in the Right view shows such a hidden feature. Students will practice including hidden lines later on, but, in the beginning, have them focus on visible distinct surfaces for each of the three views.



Have a collection of small objects (pen, can, paperclip, etc.) and have each student pick one and do the three views and have a peer review their sketches. Explain that curved surfaces are not shown as curved face on (the view of the side of a can or other cylinder is a rectangle, since there are no sudden changes in distance). Did they show all visible features for each view? Did they include any that are not actually visible? Note that which view of a shape is the front is arbitrary, but you want to orient the shape with edges that are parallel or perpendicular to that view (look head on at the pen, rather than have it at an angle).

Ask partners to discuss: can they come up with any shapes (using the cubes or thinking of familiar objects) for which two of the three views are identical? What about all three views being the same? For two of three, a can has matching side and front views (if it is standing on its base). For three of three matching, a cube and sphere are the most familiar choices, but it is possible to construct as many as we like. The shape below and to the left looks like the projection on the right from all three viewpoints.



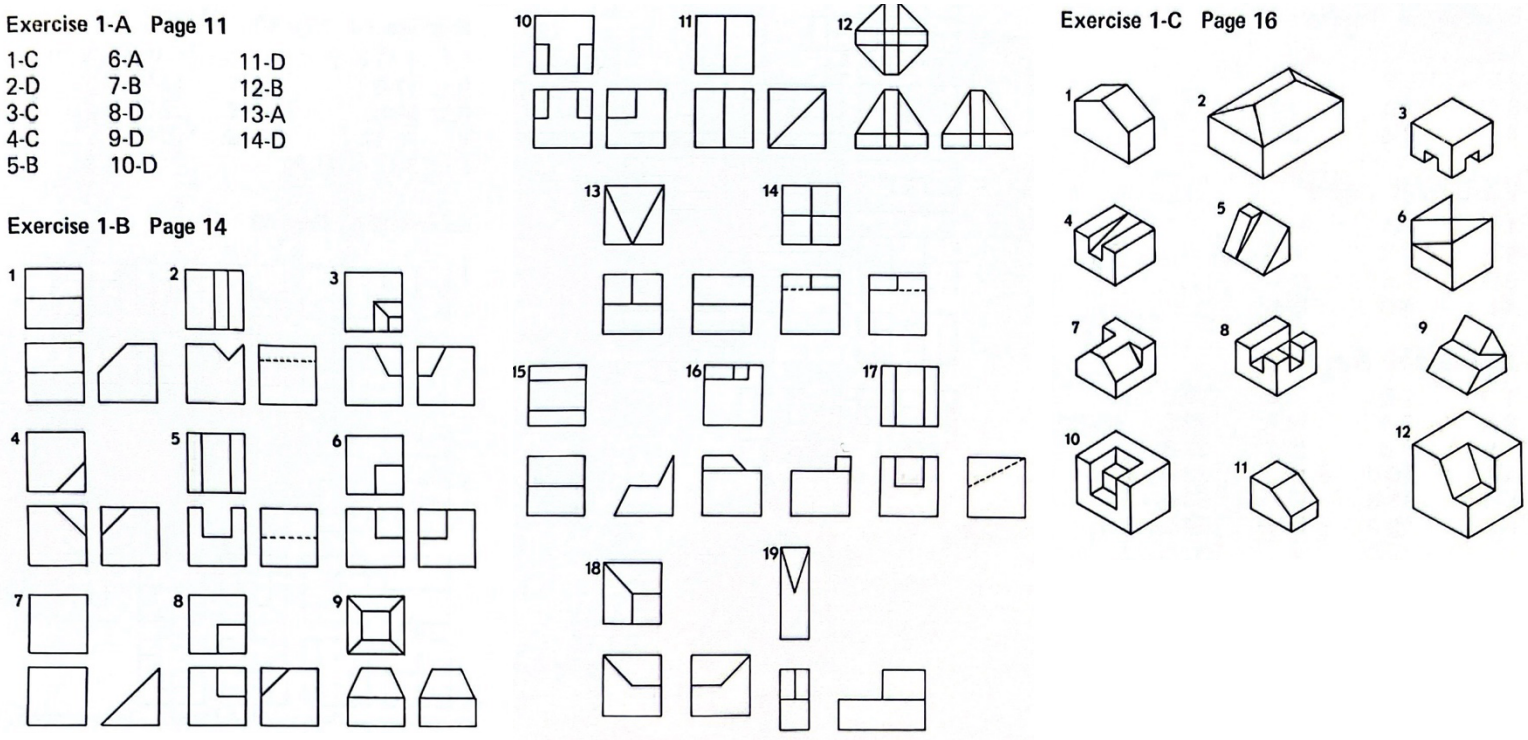
Teaser: this 3-D shape was made in OnShape by making the sketch of an octagon in the Front, Right, and Top planes and extruding each symmetrically with the second and third extrusions using the Intersect option.

Once students have had some hands- and eyes-on experiences, there are a number of great practice options both online and on paper. Here is one possible sequence using the WisWeb applets (see the "WisWeb Users Guide" below) and handouts from Seeing Shapes by the wonderful mathematics educator Ernest Ranucci. Seeing Shapes is out of print, but we are sharing samples of the exercises. These are not part of Making Math's Creative Commons license.

1. Have students work on the handout *Isometric Views*. Here are the answers:

Problem #	1	2	3	4	5	6	7	8	9	10
# Cubes	10	9	19	18	34	16	20	27	18	25
Problem #	11	12	13	14	15	16	17	18	19	
# Cubes	26	22	20	16	12	11	9	19	11	

2. Have students experiment with *Buildings with blocks* in WisWeb to learn how to build up cube structures. This practice prepares them for the other applets.
3. Have students work with *Coloring Sides 1* and *2* in WisWeb to practice identifying which faces match between 3-D and 2-D representations.
4. Have students work with *Building with side views 1* in WisWeb to match structures to given orthographic views. *Building with side views 2* includes more difficult shapes (especially given the minimum blocks goal).
5. Have students work on the handouts in *Student orthographic handouts*. Here are the solutions:



WisWeb Users Guide

This site was developed in the Netherlands with English versions of that app that sometimes don't translate every word. This guide explains how to use the site and what the Dutch words mean (your students can learn a few!).

Go to this link (https://app.dwo.nl/wisweb/?header=less&hash=#s:603083) for the English-*ish* version of the applets.

Building with blocks


Building with blocks

- This applet presents a 3-D view to the left and the goal is for students to build up a copy on the right.
- Blocks (or "blokjes") are added by clicking on a square of the base grid and then on any face of a block they have added. They can click on faces to add new blocks on top of, next to, or below pre-existing ones.
- It is easy to accidentally add a block in the wrong place. To remove a block, click on it and hold the click for a second and release it and the block will disappear. Blocks above will not fall.
- To rotate the main view to see faces to clock on, students can click and hold the mouse and move around and the view will spin.
- When they think they have a match, they should click on the "CONTROLEER" button and it will show a green check mark or red X.
- The "Maak leeg" button clears the 3-D world and "Maak vol" fills it up with blocks, if removing is faster than building up.
- There are 22 target shapes they can choose from at the bottom of the screen with this menu: 

. I don't think it is worth doing more than a few, but students can choose based on the level of challenge they want.

Coloring Sides 1 and 2


Coloring sides 1


Coloring sides 2

- In *Coloring Sides 1*, students are shown a 3-D form with one (or sometimes two) colored sides. They should spin the shape to make sure they find the colored sides. They then click on the matching sides in the orthographic views (with Left, Front, Right, and Top projections). *Coloring Sides 2* is the inverse operation: faces are colored in the projections and the correct face or faces of the solid have to be clicked on. As with the other applets, the "CHECK" button gives feedback. There are 20 problems in each of these activities. It may not be necessary for students to do all 20, but kids do get pretty quick at solving them.

Building with side views 1


Building with side views 1

- The views are labeled "boven" (Top or above), "voor" (Front), and "rechts" (Right). They change based on the structure the student builds in the main grid. Students should rotate their view to see how well their structure matches the sought after views. When the "check" button is clicked on, it only says if you are right (green check mark), right but using more than the minimum possible number of blocks (a faint yellow check mark), or incorrect (red X, because one or more views doesn't match). Given the goal of using as few blocks as possible, even challenge number one is tricky. My first solution (shown at left

below) used 14 blocks, but 6 of them can be removed without changing the views (thanks to the ability of blocks to float in position).